The Scalable Capital Investment Process

White Paper

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1 Overview

Scalable Capital offers intelligent and cost-efficient services for systematic wealth creation. Services that are commonly available only to large institutional investors or high-net-worth individuals. Scalable Capital’s investment process is built around the client, taking both investment goals and individual risk tolerance explicitly into account. Based on the investor’s individual profile and a carefully selected, comprehensive universe of index-based investment instruments, Scalable Capital deploys a highly innovative, rule-based algorithm to construct a tailor-made, optimal investment portfolio. All portfolios are continually monitored and rebalanced in order to keep them in line with the specific investment goals and risk limits.

1.1 The Building Blocks

The Scalable Capital investment process combines five building blocks that pave the way to optimal wealth creation:

- Compilation and ongoing adaptation of the relevant investment universe to always guarantee a comprehensive, globally diversified and cost-efficient set of investment opportunities.
- Determination of the investor profile in terms of investment goals, financial status and risk tolerance.
- Derivation of an investor-specific, optimal asset allocation.
- Continual process of monitoring and dynamically adjusting each individual portfolio in line with the investor’s risk tolerance.
- Regular review and updating of the investor profiles.

All these building blocks have been carefully designed and tuned to one another so that they form a comprehensive and effective wealth creation process.

1.2 The Process

The investment process of Scalable Capital aims to provide globally diversified portfolio allocations which are tailored to the individual needs and preferences of each client. Asset weights relative to market capitalizations serve as a starting point for this process, as market capitalization weights represent average investor allocations. Furthermore, market capitalization weights are well founded by the theoretical finance literature. See Section 4.2.1 for an in-depth discussion of the central model in this regard, the Capital Asset Pricing Model (CAPM). Due to regional differences in investor perspectives and currency risks, global market capitalization weights are adjusted to more closely match the investor’s perspective and reduce foreign currency exposures. These adjusted market capitalization weights form the starting point of Scalable Capital’s investment process.

The investor’s individual profile and the characteristics of the instruments in the investment universe are the central inputs to Scalable Capital’s investment process which is summarised in Figure 1. Individual investment goals and risk tolerance specify the requirements the investment process has to comply with throughout the wealth creation. The current market environment, as described by the long- and short-term behaviour of the asset classes under consideration, provides the basis for statistically projecting future market behaviour and uncertainties.

Strongly believing that the creation of wealth is best accomplished by successful risk management, Scalable Capital’s asset allocation algorithm focuses on downside-
1.2 The Process

**Figure 1: The Scalable Capital wealth creation process**

**risk protection.** Once the investor’s optimal portfolio is invested, Scalable Capital will continually monitor and project its development and assess whether it is on track or not. If the projection falls outside the prescribed risk corridor, the optimisation algorithm steps in automatically and re-optimises the portfolio.

This process differs substantially from the common approaches in the industry which usually assume time-invariant risk assessments over the whole investment period. By using simple, backward-looking measures which are solely based on volatility, these approaches fail to account for risk dynamics and therefore tend to overestimate profit chances and underestimate loss potentials.

Rather than scheduling calendar-driven rebalancing (say, biannual or annual) intervals, as is common with conventional investment strategies, Scalable Capital’s rebalancing activities are mainly driven by projected downside risk of the portfolio. In addition to risk concerns, there can be other rebalancing triggers, such as violations of portfolio weight limits, the in- or outflow of funds, or changes in the investor’s profile, which are re-assessed on a regular basis.

With its permanent risk monitoring and risk control, Scalable Capital provides investment services that are rarely afforded to private investors. Even in cases where services of this sort are offered, they typically come with rather traditional assumptions and procedures – such as risk parity approaches – and are usually based on simple, backward-looking asset-specific volatility or historical simulation instead of realistically calibrated, forward-looking Monte Carlo simulations as Scalable Capital does.

In the following sections we describe and explain the major building blocks of Scalable Capital’s investment process.
2 Investment Universe

Diversification is essential for designing successful risk-controlled investment strategies and achieving attractive risk-return profiles. Scalable Capital has, therefore, defined a broad and global universe across asset classes that covers the relevant security markets. This greatly helps to control risk through diversification, while letting the investor still benefit from the asset classes’ growth potential.

2.1 Diverse Asset Classes

Scalable Capital’s investment universe comprises all liquid asset classes that can be traded in a cost-efficient manner. It includes stocks, government and corporate bonds from all relevant developed and emerging economies, covered bonds, real estate stocks, natural resources, plus money market or cash-like instruments. The composition of the investment universe is continually monitored and adapted in order to pave the way for effective wealth creation.¹

2.2 ETFs – Cost-Efficient, Passive Instruments

Rather than investing in individual instruments (i.e., stocks or bonds) belonging to an asset class, Scalable Capital invests in funds, i.e., baskets of investment instruments, which represent a specific asset class. This guarantees diversification within each asset class and, together with the already highly diversified universe, enhances overall diversification.

In order to do this at low cost, Scalable Capital selects passive index funds, so-called Exchange Traded Funds (ETFs), as well as, potentially, Exchange Traded Commodities (ETCs) as investment vehicles.

Exchange Traded Fund (ETF)

An Exchange Traded Fund (ETF) is a fund that tracks an index of assets like stocks, bonds, commodities etc. Different from conventional funds, ETFs are directly traded on securities exchanges – just like stocks. Buying an ETF means buying shares of all assets that belong to the index. ETF managers do not try to outperform the index, they simply try to match its performance in a reliable and cost-efficient manner.

Exchange Traded Funds have a number of attractive advantages over conventional mutual funds:

Passive Management: An ETF tracks a specific market index and does not try to outperform it. Actively managed funds attempt to outperform a benchmark index and often invest significant resources into research in an attempt to do so. In the medium and long run, the vast majority of actively managed funds are not able to outperform their benchmarks. This is for two reasons: for one, it is hard to consistently “outguess” the market, and secondly the expenses for research (teams) are nontrivial and add substantially to a fund’s management cost. In contrast, ETFs run on autopilot and require only minor adjustments to keep them in line

¹ The current composition of Scalable Capital’s investment universe including the specific investment instruments can be found on our website.
with the index they track. Therefore, the expenses ETFs incur are much lower. The annual management expenses of ETFs can range from about 0.1% of the investment for mainstream indices and up to about 0.75% for highly specialised and difficult to replicate indices. Management fees for actively managed funds are typically in the range of 1.5% annually, but can be considerably higher when focusing on more exotic assets.

**Simplicity:** An investment in an ETF means investing in a basket of assets with a single transaction. There is no need to make a multitude of transactions for individual stocks or bonds which create excessive burden and multiply transaction costs. Also, since they are traded on exchanges, ETFs do not have initial fees (also called “sales fees”) or exit fees as mutual funds commonly do.

**Flexibility:** The price of mutual funds is typically set once during a day, fixing the price at which fund shares can bought or sold. Like publicly traded stocks, ETFs are traded throughout the exchange’s trading hours and can be sold and bought at prices that are continuously updated during these hours.

**Transparency:** Mutual funds reveal their specific investments only very infrequently. Consequently, most of the time, it is not clear in which assets they are invested. ETFs usually track a well-defined index, whose constituents are commonly known or published by the index or ETF provider.

All in all, ETFs are cost-efficient, diversified, flexible, and easy-to-understand investment vehicles, making them ideal instruments for wealth creation. Typically, for any given asset class, there is a range of ETFs on the market, offered by competing providers. Scalable Capital applies a rigorous selection process to identify the ETFs that serve its clients best.

### 2.3 Scalable Capital’s ETF Selection Criteria

Scalable Capital applies a range of quantitative and qualitative criteria when deciding on which ETF to include or to exclude from its investment universe:

**Quantitative Criteria:**

- **How expensive?** The *Total Expense Ratio* (TER), i.e., the ratio of the fund’s total cost over its total assets, is a measure of a fund’s overall expenses. In the investor’s interest, Scalable Capital favours low-TER ETFs.

- **How liquid?** Illiquid ETFs, i.e., ETFs with low trading volume, have larger bid-ask spreads, which increase transaction costs. We focus on large (in terms of market capitalisation) and established (in terms of issuance date) ETFs which have multiple designated market makers and liquidity providers. Everything else equal, we prefer ETFs with high liquidity and low spreads.

- **How exact?** In other words, how well does the ETF track the underlying index? ETFs that follow the strategy of *full physical replication* try to invest in the very assets the index is composed of and match the index weights more or less exactly. The *representative sampling* strategy invests only in a representative subset of the index constituents. This is typically less expensive than full physical replication,
but reduces tracking precision. Scalable Capital prefers ETFs with low tracking errors over less accurate ones.

- **How diversified?** ETFs usually track broad market indices with dozens, sometimes even hundreds of different components. This ensures exposure to the fundamental factors driving returns in a specific asset class without taking excessive risk to idiosyncratic events such as bankruptcies or sovereign defaults. On the other hand, it becomes increasingly expensive to track very broad indices as they include a significant portion of less liquid components (the so-called “long tail”). Scalable Capital’s selection process balances the need for sufficient diversification with the requirement of keeping TERs low.

### Qualitative Criteria:

- **How risky?** Apart from the general market risk, induced by the ups and downs of its constituents, an ETF investment can face other types of risk. Rather than buying the underlying assets, some ETFs use synthetic replication to track an index by entering into swap or derivative agreements with third parties. In this case, an ETF is exposed to counterparty risk, because losses may arise if a counterparty does not fulfill its part of the deal. Other ETFs engage in securities lending which might boost investor returns and managers’ revenues but also introduces additional risks.

  Everything else equal, Scalable Capital favours ETFs that have, by construction and management practices, low risk profiles.

- **How tax-friendly?** The design of an ETF can have tax implications for the investor. As tax legislation varies from country to country, Scalable Capital takes transparency and simplicity with respect to taxation into account when determining or adapting a country’s ETF universe.

- **How convertible?** ETFs are being issued in many different currencies and listed on many different markets. Scalable Capital favours instruments which trade in the investor’s home currency to mitigate foreign currency conversion costs.

- **How fractional?** In some jurisdictions, ETFs can only be bought and sold in integer quantities. Therefore, the price of an ETF determines the minimum increment when rebalancing the portfolio. The smaller the price is, the more accurate the adjustment. Everything else equal, Scalable prefers ETFs with low absolute prices.

Typically, there is not one ETF in a given peer group that dominates the others with respect to all of the above criteria. Scalable Capital tries to find the best balance when selecting ETFs to represent an asset class. For each country in which Scalable Capital operates, the ETF selection will change over time as new ETFs are issued or the attributes of existing ones change. Scalable Capital’s website provides up-to-date information about the current set of instruments making up the country-specific investment.

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2 Such risks arise when a non-synthetic ETF lends out (typically only a small fraction of its) assets against a fee. To avert the counterparty risk, the borrower generally posts a cash collateral which will be liquidated if a borrower does not return the assets. Two loss scenarios can arise: first, although the collateral is regularly adjusted, at liquidation it may fall below the value of the assets lent out; second, the ETF company itself may invest the cash collateral in a money market fund that could lose money.
3 Risk Measurement and Risk Classification

Investment goals and risk tolerance are both crucial inputs for deriving a client’s appropriate asset allocation. While it is reasonable to assume that investors have no desire to limit profits on their investments, their willingness or ability to bear losses is clearly limited. With this in mind, Scalable Capital treats an investor’s risk tolerance as a constraint that the asset allocation has to satisfy and derives the most attractive portfolio among all risk-compatible allocations.

This requires a sensible and reliable measure of investment risk and comprehension of this by the investor. Since investors associate losses rather than price fluctuations with risk, Scalable Capital focuses particularly on the use of downside-risk measures. There are several measures to choose from. Therefore, we will first introduce the different risk measures, discuss their advantages and disadvantages and their role in Scalable Capital’s investment process, before explaining the underlying risk classification scheme.

3.1 Risk Measurement

3.1.1 Alternative Concepts for Risk Measurement

Before assigning an investor to a specific risk category, the question of how to measure investment risk arises. Conventional portfolio management approaches are based on mean-variance optimisation and use the statistical concept of standard deviation (also called volatility) to measure investment risk.

### Standard deviation, volatility and variance

The *standard deviation* or *volatility* is a statistical measure of dispersion and is defined by the square root of the variance. The *variance* is defined as the expected squared deviation of the return on an investment from its expected return.

The use of standard deviation or volatility has, however, serious drawbacks:

- Although it is a commonly used statistical measure of dispersion, volatility is difficult to interpret, even for professional investors. It is therefore not the right tool for assessing risk preferences when communicating with clients.

- Even if we know the volatility of an investment, it is not clear how to infer the actual loss potential from this information. Especially over longer horizons, allocation strategies can lead to the same volatility but, at the same time, to significantly different loss potentials (measured for example by Value-at-Risk or Maximum Drawdown).

In view of these deficits, Scalable Capital’s investment process takes alternative and more appropriate risk measures into account. These measures capture, in one way or another, the loss potential or downside risk of a portfolio rather than simply its up-and-downside variations. This allows to emphasize the management of loss potentials and to account for the whole history of realised evolutions. A commonly used measure for downside risk is called *Value-at-Risk* (VaR). It has the advantages that it is easily interpretable and that it only considers loss potential. The VaR is therefore widely used in the regulation of financial institutions and has emerged as the industry standard in institutional investing. Another measure, that is closely related to VaR, is the *Expected Shortfall* (ES). ES gives the expected loss conditional on a VaR violation. Finally, the
3.1 Risk Measurement

Maximum Drawdown (MDD) is a measure that focuses solely on the worst investment outcome.

### Value-at-Risk, Expected Shortfall and Maximum Drawdown

**Value-at-Risk (VaR)** is a downside risk measure for an investment. For a given holding period, the VaR at the \( p\% \) confidence level indicates the potential loss at the end of the holding period that, with a probability of \( p\% \), is not to be exceeded. From a statistical viewpoint, the \( p\%-VaR \) corresponds to the \((100-p\%)\%-quantile\) of an investment’s return distribution. The **Expected Shortfall (ES)** builds upon VaR and is defined as the loss that is to be expected in those cases where the VaR limit is violated. The **Maximum Drawdown (MDD)** is given by the worst outcome that is to be expected.

Each of these risk measures exhibits advantages and disadvantages which are discussed below in order to show their appropriateness or inappropriateness for use in the investment process.

#### 3.1.2 VaR, ES or MDD?

The suitability of a risk measure for managing a portfolio according to the client’s risk specification depends on various criteria. Important criteria are:

- Easy to comprehend for investors
- Suitable for the chosen investment instrument
- Suitable for backtesting and empirical model validation
- Robustness in terms of computation and prediction
- Computational complexity

The following discussion shows that all three risk measures VaR, ES and MDD perform differently when taking these criteria into account.

Considering **comprehension**, MDD comes off well, since “maximal loss” represents the largest possible loss for a given period of time which will never be breached. One can say that the MDD is a limit case of VaR as the confidence level approaches 100%. Hence, the VaR corresponds to the MDD concept but uses a 95% probability instead of a 100%. ES is a conditional expected value: the expected loss under the condition of exceeding a VaR-value with a given confidence level. The interpretation of a conditional expected value is not trivial especially when applied to events that occur with small probability, i.e., when operating with high confidence levels. In addition, it is hard for an investor to assess whether or not an observed loss is in line with a given ES-target.

**Its suitability for the asset class under consideration** is relevant when choosing a risk measure. An important criterion is whether or not it can handle high-risk assets in a portfolio. A high-risk investment is characterised by having a fat-tailed return distribution. Statistically, fat-tailedness can be measured in terms of the so-called tail-index. The lower the tail-index, the higher the possibility of extreme events. Although ES is attractive in the sense that it looks beyond VaR, it has the disadvantage that it does not exist and, thus, cannot be calculated when an investment has a tail-index of one or below. In contrast, VaR and MDD always exist regardless of the underlying

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3 See, for example, the explanations on Wikipedia on [https://en.wikipedia.org/wiki/Conditional_expectation](https://en.wikipedia.org/wiki/Conditional_expectation).
3.1 Risk Measurement

distribution. In order to circumvent this ES deficiency, a common strategy is to truncate price jumps that exceed some – often arbitrarily – chosen level or to assume that returns beyond that level follow again a thin-tailed distribution. So-called “Truncated Levy Flight” or “Tempered Stable” models fall into that category. Clearly, such approaches are questionable as they assume that moderate price changes are fat-tailed whilst larger ones follow a normal or thin-tailed distribution.

In addition, the suitability of a risk measure may also depend on the types of instruments making up the investment universe. In the case of non-linear⁴ instruments (like Standard-, Barrier-, Knockout-, Lookback- etc.) options, a VaR-measure might not be sub-additive. Subadditivity is a theoretically interesting property when aggregating risks. There, subadditivity precludes a potential overestimation of the aggregated risk.

In other words: Subadditivity prevents an exaggeration of portfolio risks that arises when being overly cautious in assessing risk reduction from diversification. ES and MDD are always subadditive regardless of the nature of the universe. However, subadditivity does not need to hold true, especially when dealing with non-linear investments. In cases where the tail-index is one or less, the aggregation will increase rather than reduce overall risk. In contrast to ES, VaR is capable of producing “superadditivity” (cf. Heyde, Kou und Peng, 2009; Ibragimov, 2009).

Since the investment universe of Scalable Capital consists only of ETFs and does not contain complex, non-linear investment vehicle, the possibility of inappropriate additivity behaviour when using VaR does not arise. This is also demonstrated in a study by Danielsson et al. (2005). In a linear investment universe, ES turns out to be a redundant measure as it moves proportional with VaR. In other words, consistent use of VaR or ES will lead to equivalent allocation decisions when dealing with linear portfolios.

One of the most important criteria leads to the question to which extent the risk measure allows for backtesting and empirical model validation. The quality of a risk model as well as the effectiveness and robustness of an allocation model can, ex-ante, only be reliably assessed by the use of (retrogressive) backtests and empirical validation. A severe disadvantage of ES, compared to VaR, is that it lacks so-called “elicitability” making proper validation practically impossible. This implies that an investor has to “blindly” trust the risk model and learns about its adequacy afterwards. According to Davies (2014): “... verifying the validity of mean-based [ES] estimates is essentially more problematic than the same problem for quantile-based [VaR] statistics”⁶. To handle these difficulties with model validation, Emmer, Kratz and Tasche (2015) propose to approximate backtesting of ES-based models by deriving VaR estimates for different

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⁴ A financial instrument is linear, when the returns are linearly dependent on the underlying risk driver, as it is the case for stocks, for example.

⁵ Dhaene, Goovaerts and Kaas (2003) as well as Heyde, Kou and Peng (2009) and others doubt the reasonableness of subadditivity for risk measures. The following example, adapted from Dhaene, Goovaerts and Kaas (2003), outlines this process. In the case of risk disaggregation, subadditivity implies that when risks are disaggregated, the sum of partial risks cannot be smaller compared to the overall risk before the split. Let us assume a bank in financial distress with a total loss incoming. The banking supervision authority arranges a division into a “sickish bad bank” and a “healthy good bank”. If the total loss still threatens the bad bank but not the good bank anymore, the sum of partial risks is smaller than the risk before the division. A subadditive risk measure is unable to model this superadditive situation.

⁶ Due to this fact, Paul Embrechts, a leading financial risk-researcher, comes to the conclusion: “If you held a gun to my head and said: ‘We have to decide by the end of the day if Basel 3.5 should move to ES, with everything we know now, or do we stick with VaR’, I would say: ‘Stick with VaR’.” (Risk.net, March 2013, http://www.risk.net/risk-magazine/news/2253463/mooted-var-substitute-cannot-be-backtested-says-top-quant).
3.1 Risk Measurement

certainty levels. Here, the question arises whether VaR-based methods shouldn’t be the preferred choice in the first place.

Only forward-looking risk management can be effective. Therefore it is important that risk measures are robust with respect to computation and prediction. Robustness is especially problematic for risk measures like ES and MDD since they need extreme-tail information. By definition, tails describe the far ends of a distribution. When considering losses, the tail is given by the single most extreme loss observed. Since statistical analyses cannot be conducted with a single observation, one needs to “improvise”. Two strategies can be adopted to avoid this problem: Looking at more losses than just the most extreme loss, or postulating assumptions about the distributional behaviour in the tail area. The problem with the first strategy is that, although risk estimates can be stabilised, one tends to lose information about the actual distributional properties in the tails. The more non-tail observations are considered the less one learns about the tails and the more risk-measure estimates become distorted. On the other hand, using only a few, extreme observations induces less robustness because estimates become rather erratic. The second strategy suffers from high sensitivity with respect to the distributional assumption for the tail. It is the belief of the modeller, and not actual data that predominantly determine risk assessment. The drawbacks of both strategies speak against the usage of ES and MDD as risk measures: Stable ES and MDD assessments tend to either rely on biased, data-based estimation procedures or on restrictive and often highly unrealistic assumptions like thin-tails and temporal independence of returns. Hence, both risk measures are not well suited for forward-looking risk and portfolio management. This conclusion is supported by Davis (2014): “... significant conditions must be imposed to secure consistency of mean-type [ES] estimates, in contrast to the situation for quantile [VaR] estimates ... where almost no conditions are imposed”.

Heyde, Kou and Peng (2009) suggest that the “Tail Conditional Median” (TCM) is a more resilient risk measure that can look beyond VaR. It corresponds to the ES but uses the conditional median instead of the conditional mean and can also be referred to as “median shortfall”. The median is less sensitive to outliers compared to the mean. However, Heyde, Kou and Peng (2009) emphasise that for linear investments TCM and VaR are equivalent. TCM at confidence level of 100-α% amounts to VaR of 100-α%. If you are looking for a robust risk measure for a linear portfolio that reflects tail information and is robust, you should use VaR with an appropriately chosen confidence level.

In practical applications, computational complexity is desirable, since it simplifies the numerical derivation of optimal portfolios and helps to avoid suboptimal solutions. The difficulty of the MDD is its extreme sensitivity, which upstages numerical aspects and thereby prevents reliable portfolio optimisation. Since VaR isn’t necessarily a convex function of the portfolio weights, VaR-based portfolio optimisation can be burdensome. However, modern computational procedures now allow the calculation of robust solutions for non-convex optimisation problems. For most applications, disregarding perhaps high-frequency trading or very large portfolios, the additional time and computational effort required for VaR-based optimisation may be well worth it.

Table 1 summarises the above discussion on the suitability of risk measures and reveals that no single measure dominates with respect to all criteria - each measure has certain

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Chekhlov, Uryasev and Zabarankin (2005) state: “... however attractive the MaxDD measure is, the solutions produced using this measure in portfolio optimisation may have a significant statistical error because the decision is based on a single observation of the maximal loss.”.
3.2 Risk Classification

Table 1: Criteria for the suitability of risk measures

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<th>VaR</th>
<th>ES (CVaR)</th>
<th>MDD</th>
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<tr>
<td>Simple traceability</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Suitable for high risk investments</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Suitable for linear/ non-linear investments</td>
<td>Yes/No</td>
<td>Yes/Yes</td>
<td>Yes/Yes</td>
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<tr>
<td>Suitable for backtesting and model validation</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Robust calculability and predictability</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
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<tr>
<td>Computational simplicity</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
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advantages and disadvantages. As explained next, even though the overall advantages of VaR clearly prevail, Scalable Capital’s investment process makes use of all three risk measures.

3.1.3 Scalable Capital’s Selection

Since the different risk measures, VaR, ES and MDD, complement each other regarding their explanatory content, all three enter Scalable Capital’s investment process. However, given the differences in comprehensibleness for investors and the usefulness for backward- and forward-looking analyses, these risk measures are used in different stages throughout the investment process. All three measures, as well as other indicators, play a role when it comes to backward-looking analyses and ongoing monitoring processes. Because MDD is not suited for forward-looking analyses, it plays no role in controlling portfolios based on risk projections.

Given a linear investment universe and proper usage, VaR and ES lead to equivalent asset allocations. Nonetheless, VaR is easier to understand than ES and hence plays the key part when it comes to the risk classification of investors. Once an investor’s appropriate VaR level has been chosen, Scalable Capital optimises, monitors and controls the portfolio with the aim to stay close to that pre-specified VaR target.

The VaR is defined for the confidence level of 95% and an investment horizon of one year. Therefore, it can be viewed as the percentage loss over the next year that should not be exceeded with a probability of 5% or, in other words, in one out of 20 years the portfolio is expected to lose more than the specified VaR target.

3.2 Risk Classification

The initial step in Scalable Capital’s wealth creation process is to get to know the investor. This is not only a legal requirement, but also essential for deriving an investment strategy that is optimal for the client. The investor profile is obtained online, through a sequence of straightforward questions. The information is processed in real-time so that the investor learns right away about his or her proposed risk classification. If the investor chooses the suggested – or a more conservative – category, the current optimal asset allocation is depicted.

3.2.1 Status and Investment Goals

The online questionnaire begins with questions about the investor’s personal and financial situation. Investment horizon, previous investment experience, and the understanding of specific investment instruments and their risk-return characteristics are queried. Next, risk-return preferences will be elicited based on the investor’s tolerance for loss
and their goals for the investment. A consistency check runs in the background to assess whether or not Scalable Capital’s investment services are suitable for this particular investor. If this is not the case, the applicant will be informed about this and the onboarding process will be stopped.

### 3.2.2 VaR-based Risk Classification

Based on the answers to the questionnaire, Scalable Capital assesses for each investor his or her individual risk tolerance and determines the corresponding maximum VaR-limit. This limit reflects the investor’s risk capacity and risk attitude. Scalable Capital offers altogether 23 risk categories, ranging from 3 to 25, associated with the annual 95%-VaR-limits 3% to 25%. For example, an investor choosing risk category 12 wants to limit downside risk so that an annual decline of more than 12% should occur on average only once in 20 years. The risk projection of each client portfolio needs to be compatible with this VaR-limit at all times.

When going through this elicitation process, the investor is assisted with graphical displays of potential profit-loss ranges, which reflect the range of likely outcomes of investments associated with different risk classes (i.e., VaR-limits). The set of likely scenarios associated with a risk class is summarised in terms of the expected median scenario and the central 90% confidence band.\(^8\) All these scenario summaries are exemplary depictions, as they are based on historically observed risk-return relationships.

Having completed the survey, Scalable Capital proposes the appropriate risk category. If the investor is not comfortable with this limit, he or she can choose a lower, but not a higher, risk limit. After the risk category has been selected, the portfolio composition that corresponds to that risk class will be shown.

\(^8\) The bandwidths are defined by the 95%- and the 5%-quantiles of the scenarios.
4 Portfolio Optimisation – Motivation and Process

Given a carefully constructed investment universe as well as the individual risk classification, the question of how to derive a portfolio that best serves the investor leads to the central building block of Scalable Capital’s investment process. The objective is to generate the best long-term return without violating the investor’s risk limits. To achieve this, optimal portfolio allocations are derived by systematically combining ETFs from the Scalable Capital investment universe.

What does “optimal” mean? Clearly, the typical investor prefers high-yielding over low-yielding investments. Unfortunately, investments with high return prospects tend to also have a higher risk of generating losses. This risk-return predicament forces the investor to find the “right balance” between risk and return prospects. A portfolio or asset allocation is deemed *optimal*, if its expected return is the highest of all permissible allocations that do not exceed the investor’s risk limit. While portfolio risk can be forecasted rather well, there is a consensus in the financial literature that short-term expected returns are almost unpredictable. A favourable risk/return ratio of a client portfolio can hence mainly be achieved by managing portfolio risks. Such risk management is therefore central to the Scalable Capital investment process. Essentially, the Scalable Capital investment process is driven by:

- the investor’s individual profile, reflecting investment goals and risk tolerance,
- representative portfolios for individual asset classes, i.e., market-capitalization weighted portfolios with adjustments for investors’ regional perspectives and home currency,
- the current market conditions, i.e., the risk-return prospects of the assets in the investment universe as well as their potential for risk diversification, and
- the implicit and explicit costs from transacting and holding investment vehicles.

Based on this information, Scalable Capital’s optimisation engine, sketched in Figure 2, derives for each investor the individually optimal asset allocation.

![Figure 2: The Scalable Capital portfolio optimisation process](image)

Since avoiding large losses is crucial for successful wealth creation, Scalable Capital’s overriding concern is to control downside risk. To accomplish this, the portfolio weights
are set such that the portfolio’s projected risk is compatible with the investor’s risk tolerance (VaR-limit). The reasons why downside-risk protection is instrumental for successful wealth creation and how Scalable Capital implements this protection will be explained in the next sections.

4.1 Conventional Mean/Variance Optimization

Most quantitative portfolio managers rely on a conventional mean-variance optimisation (MVO) for portfolio construction which was developed by Harry Markowitz in the 1950s (see Markowitz, 1952). At that time, MVO represented a major scientific breakthrough and it has become a cornerstone of modern finance. For his pioneering work, Markowitz was honoured with the Alfred Nobel Memorial Prize in 1990 and the Swedish Academy of Science wrote:

Markowitz showed that under certain given conditions, an investor’s portfolio choice can be reduced to balancing two dimensions, i.e., the expected return on the portfolio and its variance. Due to the possibility of reducing risk through diversification, the risk of the portfolio, measured as its variance, will depend not only on the individual variances of the return on different assets, but also on the pairwise covariances of all assets.

Undoubtedly, Markowitz’ contribution represented a major breakthrough at the time, and the Nobel Memorial Price was well deserved. A direct implementation however suffers from serious drawbacks. The major weakness of the approach is that distributional moments of returns are assumed to be known in the Markowitz framework even though they need to be estimated from data in an actual application. This means that estimation error is entering the portfolio decision without any additional adjustment in a Markowitz optimisation. Assets which seem favourable simply due to estimation error in the respective return moments are thus excessively overweighted in the resulting portfolio allocation. This leads to suboptimal investment decisions and, due to the imprecisely estimated inputs, extremely concentrated portfolios (see for example Scherer, 2002).

Covariance and correlation

Both covariance and correlation try to measure the dependence or “synchronicity” of the behaviour of two assets. The covariance looks at the simultaneous behaviour of the two assets’ returns in terms of deviations from their respective means. The expected value of the product of these two deviations defines the covariance. The correlation is a standardised version of the covariance (assuming values between +1 and −1) that is obtained by dividing the covariance by the two return volatilities.

A positive correlation indicates that both assets tend to jointly out- or underperform their mean; a negative correlation indicates that the assets tend to move in opposite directions from their respective means. These tendencies become stronger as the correlation approaches ±1.

Additionally, if a Markowitz optimisation is carried out in a static manner without dynamic readjustments, a further caveat results. This caveat was also addressed in the statement of the Swedish Academy of Science which emphasized that “the approach

4.2 Portfolio Diversification

requires certain given conditions. One sufficient condition for Markowitz optimisation leading to optimal allocations is that investor preferences can be fully described by portfolio mean and variance. This is however in direct contrast to the deficits of volatility as a single risk measure which are discussed in Chapter 3.1. Alternatively, Markowitz optimization also generates optimal allocations for given distributional parameters if the following conditions are satisfied:

- Assets behave symmetrically in the sense that they have identical up- and downside potentials.
- Returns of different assets are linearly related to each other. The synchronicity with which asset prices move is constant regardless of whether we are, say, in a bear or bull market.

Numerous empirical studies, starting with the pioneering works of Benoit Mandelbrot and Eugene Fama in the 1960s, have shown that these assumptions underlying the Markowitz approach are critically violated in practice. Without explicitly taking into account these violations, MVO has fundamental practical shortcomings, which often lead to inadequate asset allocations and hence severe drawbacks for clients. The Scalable Capital investment process therefore builds on more realistic capital market assumptions and takes additional risk measures, which are more complex than volatility, into account. Risks are dynamically assessed and managed to account for time-varying loss potentials.

4.2 Portfolio Diversification

MVO only leads to optimal allocations if the inputs to the optimisation are precisely known. This is not the case in actual applications in which means and variances have be estimated from available data. The introduced estimation error often leads to extreme portfolio allocations in a straightforward application of MVO. Thus, the process has to be adjusted such that a certain robustness against estimation errors in the inputs is guaranteed. This can be achieved by introducing a preference for well diversified portfolios into the optimisation objective. The Scalable Capital investment process favours well diversified, market-capitalization weighted allocations within each asset class. Market-capitalization weights can be interpreted as average investments of all global investors and hence reflect the portfolio decision of an average representative investor. To better reflect regional investor preferences driven by currency concerns, pure market-capitalization weights are adjusted to comply with representative investors of equal home currency. These well-diversified representative allocations serve as starting point in the optimization process and help to significantly reduce the influence of estimation errors and to realise diversification potentials in each asset class. The overall starting point of market-capitalization weighted allocations is well-justified by the theoretical literature and the findings of the Capital Asset Pricing Model, where market-capitalization weights arise as the equilibrium outcome. This model, which is one of the cornerstones of financial research, is discussed in the next section.

10 See for example Mittnik et al. (2000) and Rachev and Mittnik (2000).
11 See Mandelbrot (1963) and Fama (1965).
4.2 Portfolio Diversification

4.2.1 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM), which was developed on basis of the insights of Markowitz (see Sharpe, 1964, and Lintner, 1965), provides the central argument for a risk-return relationship in financial markets. The CAPM states that expected returns of assets are proportional to the asset’s risk contribution to an optimally diversified portfolio. Mathematically, the following relationship between expected returns of a certain asset \( \mu_i \) and the expected return of an optimally diversified portfolio \( \mu_m \) results

\[
\mu_i = r_f + \beta_i (\mu_m - r_f)
\]

with

\[
\beta_i = \frac{\text{Cov}(r_m, r_i)}{\text{Var}(r_m)}.
\]

The variable \( \beta_i \) gives the sensitivity of the returns of the asset \( r_i \) with respect to the returns of the optimally diversified portfolio \( r_m \) and thus reflects the risk contribution of asset \( i \) to such a portfolio. Therefore, if the risk contribution of asset \( i \) increases, for example because the asset volatility increases for a given correlation, \( \beta_i \) increases respectively.

Hence, simplistically speaking, the CAPM states that, in a market in which all investors dislike high risks and appreciate high returns, non-diversifiable risk is compensated by expected returns. This is, from the perspective of the theoretical capital markets literature, the central argument for a risk-return relationship.

From this theoretical insight, and given some additional assumptions, it can be deduced that all market participants want to hold the same portfolio, the so-called market portfolio. To account for their individual risk preferences, they will simply adjust the fraction of their wealth which they invest into this market portfolio instead of allocating it to a risk-free investment with interest rate \( r_f \). It follows from this that the observed market-capitalizations of assets should be the weights of this, optimally diversified, market portfolio. This line of reasoning motivates and explains market-capitalization weighted allocations within most of the popular financial market indices and is the main argument for passive investment philosophies.

Scalable Capital uses the insights of the CAPM as a starting point for a global, market-capitalization based diversification strategy. The relative market-capitalizations also describe the average allocation that is held over all market participants. Every deviation from such an average allocation, often also called the allocation of a “representative investor”, must be justified against the theoretical arguments of the CAPM. However, a divergence between the theoretical implications of the CAPM and empirical facts can make a deviation from market-capitalization based portfolio weights necessary. Scalable Capital thus considers not only the theoretical insights of the CAPM but also the current empirical facts to derive optimal allocations for each client.

4.2.2 Constrained Portfolio Weights

A scientifically accepted approach to robust portfolio allocation is the introduction of portfolio weight constraints which enforce diversified portfolios. The Scalable Capital investment process includes lower and upper constraints on groups of assets. Such a group is always made of instruments which represent the same broad asset class. Such bounds
4.3 Predictability and Modelling of Returns

are common practice in asset management as they rule out portfolios which are highly concentrated in specific asset classes and thus ensure a certain level of diversification. The exact constraint levels vary over the risk categories. Additionally, Scalable Capital enforces relative constraints within each asset class to ensure well diversified allocations on an asset class level. All constraints are constructed such that the resulting allocations are reasonably close to the allocation of a representative investor with the same home currency. Optimal allocations for investors with different home currencies differ due to regional preferences and exposure to foreign exchange rate risk. A representative allocation for an investor with a specific home currency is therefore a globally diversified, market-capitalization weighted allocation that, in most cases, underweights assets that are denominated in foreign currencies to reduce exchange rate risk. By constructing portfolio allocations relative to the allocation of a representative investor, Scalable Capital adopts a passive investment strategy that reduces exchange rate risk for its clients.

Besides the economic arguments, the introduction of portfolio weight constraints is also beneficial from a statistical perspective. Weight constraints ensure a certain level of diversification and thus achieve a robustness against estimation error. These constraints therefore directly address the major drawback of conventional MVO and ensure an adequate level of robustness against flawed inputs. The appropriateness of all constraints is continuously assessed and adjusted whenever necessary.

4.3 Predictability and Modelling of Returns

Traditionally, asset managers see their main job as foreseeing future price movements of asset classes and individual assets, and they spend substantial resources on “research” to predict price trends. To do so, they resort to a range of different recipes using, for example, technical, fundamental, contrarian and various other philosophies. Clearly, being able to always accurately predict asset prices guarantees outperformance of the market. Furthermore, with accurate price predictions all market risk would be eliminated, and an investor could make unlimited, risk-free profits.

At Scalable Capital we do not believe in such paradise-like conditions and follow the overwhelming consensus from academic research, which suggests that financial markets are largely price-efficient. In other words: We take the view that the potential for outright, systematic price predictability for liquid assets is very limited.

Using a very long history of the Dow Jones Industrial Average the following analysis underpins this view. It also demonstrates that the prediction of market risk is much more promising than trying to directly predict price movements.

Example 1 (Return and risk predictability) Let us look at the Dow Jones Industrial Average (DJIA) index, one of the longest financial return series available, to examine return and risk predictability. Figure 3 shows the weekly DJIA index from 1896 to 2015. If future returns were predictable from historical returns, there should be temporal dependence between successive return observations. A common tool to assess the presence of temporal dependence in time series data is the so-called autocorrelation function (ACF). The ACF is a form of “memory function” that measures the strength and duration of (Pearson) correlation between increasingly distant return observations and, thus, is a measure of (linear) predictability.

The top panel in Figure 4 plots the weekly returns of the DJIA (left) and their ACF for a time horizon of up to one year or 250 trading days (right). The ACF shows that there is practically no linear predictability in the return series. All autocorrelations are near zero and mostly statistically insignificant (i.e., inside the 95%-confidence band indicated
4.3 Predictability and Modelling of Returns

Figure 3: Dow Jones Industrial Average index, weekly data from 1896 to 2015.

by the two horizontal red lines). From this we can conclude that past returns provide little or no information about future return behaviour.

To further analyse the question of predictability, let us decompose the return series into two components: the direction of the returns, expressed by a sequence of ±1-values (or, in case of no change, 0); and the size of the return, i.e., the absolute value of the return. Expressed differently, an observed return can be written as the product

\[ \text{Return} = \text{Return Direction} \times \text{Return Size} \]

or, mathematically, as \( r_t = \text{sgn}(r_t) \times |r_t| \). The time series of the directional component simply tells us whether the DJIA has moved up or down on a particular day; and the size component essentially reflects the risk of an investment in the index: A large range indicates a large loss (or gain) potential.

If we can (linearly) predict the direction of the DJIA, autocorrelations of the directional component should be large and statistically significant. The center panel in Figure 4 shows the time series of past DJIA directions (left) and the corresponding ACF (right). It turns out that the directional ACF behaves very much like that of the returns: All autocorrelations are near zero and mostly insignificant. Thus, past directional behaviour does not tell us much about future up or down movements. This stands in stark contrast to the behaviour of the risk component, shown in the bottom panel of Figure 4: Autocorrelations are large and statistically significant over a horizon of more than a year (250 trading days), indicating that there is substantial predictability in risk.

The fact that all autocorrelations are positive implies that there is risk clustering in the DJIA: Periods of high (low) volatility tend to be followed by periods of high (low) volatility.

This example illustrates that it is very hard, if not impossible, to come up with reliable asset-price predictions. The problem lies in the prediction of the direction of future price movements. Forecasting financial market risk, on the other hand, is much more promising given the strong empirical evidence for (linear) predictability. The econometrician Robert F. Engle received the Nobel Memorial Prize for developing a concept for
Figure 4: Linear predictability of returns on the DJIA index (top panel) and its components, namely, the return direction (center panel) and the return size (bottom panel). The data is shown on the left and the corresponding “memory functions” on the right, where bars ending inside the two red lines are statistically insignificant (noise) and those ending outside indicate significant linear predictability. The empirical memory functions suggest that neither returns themselves (top, right) nor return directions (center, right) can be predicted. There is, however, substantial predictability in risk (bottom, right).

predicting financial-market risk (see Engle, 1982). Scalable Capital exploits the fact that there is more predictability in market risk than in market direction and uses advanced methods that build on Engle’s original work.

However, in the portfolio context, risks are not only driven by the risks of the individual assets but also by the dependence structure of the instruments. The overall risk therefore depends on how the individual asset returns are related to each other. Dependency measures allow to describe the degree of synchronicity in the movement of asset prices. Risk is however not a static concept and dependence structures change over time – stock returns for example tend to move more aligned during market downturns. This shows that the estimation of dependence structures and the subsequent risk management has to account for the empirical fact of dynamic dependencies. The optimization process of Scalable Capital therefore reevaluates dependence structures on a continuous basis.

4.4 Dynamic Risk Management: Risk-based Rebalancing

As financial markets evolve, a portfolio’s allocation can become suboptimal and, thus, ought to be rebalanced. Common portfolio rebalancing strategies are calendar-driven.
They simply re-adjust the portfolio after a fixed time interval and re-set all weights to preassigned, fixed values. Scalable Capital takes a more intelligent, event-driven approach. We continually monitor each investor’s portfolio, to make sure that it is in line with the designated risk level, that all weight limits are satisfied and that the portfolio remains optimal with regards to other key measures. If a risk projection signals a violation of the VaR-target, the portfolio will be re-optimised to ensure that it stays within the assigned risk corridor – regardless of when the last rebalancing took place. This allows Scalable Capital to immediately respond to changes in market conditions without being subjected to calendrical rituals.

Expectations about how specific portfolios will evolve must be adapted whenever market situations change. In such a case, Scalable Capitals highest priority is to ensure, that the risk limit of each client is adhered. The risk limit is given by the Value-at-Risk that is associated with the client’s risk category. It must be fulfilled over the horizon of one year and thus over the aggregate of potentially multiple rebalancing events. The portfolio weights are adjusted to a less risky allocation whenever current portfolio weights cannot rule out that the risk limit of the respective client is violated. If current weights are however too conservative due to a decrease in current market risks, portfolio weights are adjusted to reflect a more risky allocation. This allows to capture profit potentials in upward moving markets. During this process, weight constraints are imposed to ensure a well-diversified portfolio in all market situations. Besides risk-based reasons, rebalancing is also triggered whenever assets approach their weight constraints, the portfolio appears sub-optimal with regards to other key measures, or if deposits or withdrawals are conducted by the client.

The concept of risk-based rebalancing is illustrated by Figure 5. The left panel shows an exemplary risk projection which is generated during the monitoring process. It can be seen that the projected loss potential (red curve) violates the one-year VaR limit (green region). Such a situation automatically triggers an additional optimisation which adjusts the portfolio allocation towards a more conservative allocation. The risk projection for the adjusted portfolio displayed in the right panel is again aligned with the risk profile and risk limit of the client. As expected, return potentials decrease for such a, more conservative, allocation. This can be inferred from the lower level of the blue curve and the lower level of the average performance projection (dashed line) in the right panel.
4.4 Dynamic Risk Management: Risk-based Rebalancing

4.4.1 Risk Projections

Scalable Capital uses methods from financial econometrics to generate risk projections. Since it cannot be expected that a single approach will dominate all others in all market situations, Scalable Capital employs a range of alternative strategies for VaR-predictions.\textsuperscript{12} The majority of them is based on Monte Carlo simulations, in which we generate up to $S = 10,000$ or more scenarios for possible future asset price movements.

Scalable Capital’s Monte Carlo approaches have in common that they draw the scenarios from a model of the form

$$ r_{t+1} = \mu_t(r_t, r_{t-1}, r_{t-2}, \ldots) + \varepsilon_t(r_t, r_{t-1}, r_{t-2}, \ldots), $$

where $r_t = (r_{1t}, r_{2t}, \ldots, r_{Nt})'$ denotes the vector of returns in period $t$ of the $N$ assets in the investment universe; and $\mu_t(\cdot)$ and $\varepsilon_t(\cdot)$ are the location function and random dispersion-from-location function, respectively, both of which depend on current and past market conditions. Let $\hat{\mu}_t(\cdot)$ and $\hat{\varepsilon}_t(\cdot)$ denote the empirically estimated versions of these functions, then, scenario $s$ ($s = 1, 2, \ldots, S$), originating in period $T$, can be derived recursively via

$$ r_{s,T+1} = \hat{\mu}_T(r_T, r_{T-1}, \ldots) + \hat{\varepsilon}_{s,T}(r_T, r_{T-1}, \ldots) $$
$$ r_{s,T+2} = \hat{\mu}_T(r_{s,T+1}, r_T, \ldots) + \hat{\varepsilon}_{s,T}(r_{s,T+1}, r_T, \ldots) $$
$$ r_{s,T+3} = \hat{\mu}_T(r_{s,T+2}, r_{s,T+1}, \ldots) + \hat{\varepsilon}_{s,T}(r_{s,T+2}, r_{s,T+1}, \ldots) $$
$$ \vdots $$
$$ r_{s,T+H} = \hat{\mu}_T(r_{s,T+H-1}, r_{s,T+H-2}, \ldots) + \hat{\varepsilon}_{s,T}(r_{s,T+H-1}, r_{s,T+H-2}, \ldots), $$

where $H$ denotes the projection horizon, i.e., the number of future periods to be projected. Applying this recursion for $S$ replications, we can assess the range of possible outcomes and, in particular, obtain the one-year VaR-projection at the 95% confidence level, given by the 5%-quantile of the $S$ one-year scenarios.

To illustrate this, let us look at the S&P 500 net total return index.\textsuperscript{13} The graphs in the top panel in Figure 6 plot the index and the daily returns from 04.01.1999 to 26.09.2014, where the index finished at 2,596 points. Employing recursion (3) to generate a one-year VaR projection with the information given on 26.09.2014, we obtain the scenarios shown on the left in the center panel in Figure 6. The histogram in the center right gives the distribution of the one-year scenarios and indicates the position of the 5%-quantile (i.e., the negative, one-year 95%-VaR), which amounts to a VaR of about 27%. The graph on the bottom left in Figure 6 shows all intermediate projections for the 5%- (red line) and 95%-quantiles (blue line). 90% of all scenarios lie within the cone formed by these two lines. Finally, the graph on the bottom right indicates the implied 90%-confidence band for the S&P 500 index level originating on 26.09.2014. According to the one-year projection, the index is expected to lie, with a probability of 95%, above 1,905 points.

VaR-projections can vary greatly as market conditions change. Figure 7 shows the 90%-confidence bands for the S&P 500 return (left graphs) and the level (right graphs) generated at four arbitrary dates. The one-year VaR-projection on 18.12.2000, shortly

\textsuperscript{12} See, for example, Mittnik and Paolella (2003) and Kuester, Mittnik and Paolella (2006) for alternative, univariate projection strategies.

\textsuperscript{13} Bloomberg ticker: SPTR500N Index. In the following we simply refer to the S&P 500 index.
Figure 6: Illustration of Scalable Capital’s VaR projection strategy. The top panel plots the index and the daily returns from 04.01.1999 to 26.09.2014. A set of possible one-year scenarios is shown in the center left. The histogram in the center right reflects the distribution of these one-year scenarios and indicates the corresponding 95%-VaR (i.e., negative 5%-quantile). The bottom left graph shows all intermediate projections for the 5%- (red line) and 95%-quantiles (blue line), forming an envelope for 90% of the scenarios. The graph on the bottom right depicts the implied 90%-confidence band for the S&P 500 projections for up to one year.
Figure 7: Relative (left) and absolute (right) S&P 500 90%-confidence bands at different dates. The lower branches of the bands correspond to 95%-VaR projections.
after the dot-com bubble began to deflate, was 34%. Towards the end of the U.S. 2007-2009 bear market, on 13.01.2009, the VaR-projection reached a very high level of 53%. The range of the corresponding confidence band, i.e., the second cone on the left in Figure 7 reflects heightening market uncertainty around that date. Shortly after that date, the S&P 500 rose significantly up to the end of 2014. In that growth period the projected market risk dropped sharply. On 24.03.2011, the VaR-projection fell to 28.5%, and on 26.09.2014 even further to 27%.

The broad variation in riskiness also applies to other asset classes. Table 2 reports VaR-projections for alternative asset classes (including bonds, stocks, real estate, and commodities) for the four very same dates considered in the preceding S&P 500 illustration. Not all asset classes move in sync. Bonds vary much less than the other asset classes – not only in absolute but also in relative terms. For most asset classes, market risk is the highest on 13.01.2009. But there are exceptions: European Corporate Bonds and Japanese Stocks peak on 18.12.2000 and 24.03.2011, respectively. These temporal variations in risk profiles clearly indicate that static allocation strategies are exposed to significant variations in loss potentials. Stabilising portfolio risks over time is a central concern of the Scalable Capital risk management that aims at a balanced tapping of the risk budget. This is achieved by adjusting the allocation in response to changes in overall risk potentials. In this process, satisfying the target risk over the horizon of one year, i.e. the target Value-at-Risk of the client, is the major priority. This does not mean that rebalancing is triggered immediately by every change in estimated risk potentials, but temporary and persistent changes in risk potentials need to be differentiated. If, for example, markets are excessively nervous and on a downwards trend for a longer period, risk management has the potential to avoid losses by reallocating capital to less risky assets. Similar to portfolio insurance strategies, risk management therefore allows to generate value for clients especially during periods in which investors are most vulnerable. It generates a certain level of safety that helps clients to continue with a predefined investment approach during crisis periods and therefore helps to avoid premature loss realisations during market downturns.

### 4.5 The Interaction Between Risk, Return and Diversification

One of the key principles of financial markets is that the return on an investment instrument and its underlying risk tend to be closely related. High return prospects are often associated with high risks. Scalable Capital acknowledges this rationale and has con-
4.5 The Interaction Between Risk, Return and Diversification

Figure 8: Average annual return and volatility levels of different asset classes from 1973–2015. Source: New Frontier Advisors LLC

structured its investment universe accordingly, covering asset classes ranging from lower-risk assets such as bonds to higher-risk assets such as emerging market stocks.

However, from an empirical perspective, the interplay between risk and return is far more complex than the statement “higher returns require higher risk” might suggest. Looking at longer time periods, we typically observe that assets with a higher average risk generate higher returns.

However, if the risk of an asset class deviates from its average or equilibrium risk level, i.e. in the presence of – positive or negative – excess risk, returns do not necessarily react proportionally. In fact, the opposite tends to happen: Periods of positive excess risk coincide with poor asset performance and negative excess risk with a strong performance.\(^\text{14}\)

In addition, excess risk tends to affect the “synchronicity” with which stock prices move. In phases of positive excess risk the degree of synchronicity typically increases. As a result, common diversification strategies fail in phases of high positive excess risk and can lead to unexpected losses, even for seemingly well-diversified portfolios.

The interaction between risk and return is illustrated in the following sections. For its investment approach, Scalable Capital allows for the complex risk-return dynamics which are observed in reality as well as the fact that the interdependences of assets generally change over time.

4.5.1 Average Risks and Average Returns

Looking at the cross-section of assets, one finds that assets averaging a higher risk over long periods also tend to yield higher returns in the long run. Figure 8 illustrates the inflation-adjusted average annual returns and risks (measured in terms of volatility) of different asset classes during 1973–2015. All bonds have a volatility below 11% and inflation-adjusted annual returns below 6%. Stocks, on the other hand, have an average

\(^{14}\) Cao and Xu (2010) show that returns of U.S. stocks behave in such a fashion.
4.5 The Interaction Between Risk, Return and Diversification

Figure 9: FTSE and FTSE-volatility (as measured by the VFTSE) from 2000 to 2016 (scaled log-scale). Source: Bloomberg

annual return of over 6%, reaching up to 11%, and their corresponding volatilities are over 15% - significantly riskier than the aforementioned bonds.

The indicated regression line in Figure 8 implies that, on average, for each percentage increase in long-term volatility, the average annual return increases by about 0.4%. A look at the development of commodities over the past four decades demonstrates the fact that “high risk equals higher returns” isn’t a universal truth. Even though commodities had a relatively high volatility of close to 20%, their average annual return was only about 3.3%.

4.5.2 Excess Risk and Return

A longitudinal investigation into how risk fluctuates over the long term for a given asset class paints a different picture of the risk-return relationship. When the risk of, say, equity steeply increases, this usually coincides with below-average or negative market performance. This is especially true when risk levels are already elevated. Figure 9 illustrates the development of the FTSE 100 volatility (measured by the VFTSE, the risk/volatility indicator of the FTSE 100) from January 2006 to March 2016 and highlights the fact that risk can vary drastically. Whilst the volatility had already increased gradually during the subprime crisis, it then rose a further fourfold as a result of the financial crash associated with the fall of Lehman Brothers in mid-September 2008. The national debt crisis of 2011 brought another rise, which increased volatility by a factor of three.

Looking at the FTSE 100 performance in those periods, it is apparent that a severe increase in risk is paralleled by longer term, dramatic market slumps. Namely, roughly 50% and 30% in the course of the banking crisis and the national debt crisis, respectively. The horizontal line in Figure 9 indicates a kind of equilibrium risk level (median) and

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clearly identifies a common theme: When the risk surpasses the median (therefore operating in positive excess risk), we generally observe a plummeting FTSE 100. However, when the excess risk is heavily negative or when the risk level plummets, we can observe that the FTSE 100 tends to increase.

The relationship of the FTSE 100 and the VFTSE, shown in Figure 9 in terms of time series plots, can also be illustrated in terms of a scatterplot as shown in Figure 10. Here too, we can see that increases in volatility are often accompanied by losses in the FTSE 100. This phenomenon is independent of the respective level of the FTSE 100 as is indicated by the quasi-parallel stratification of the data.

There thus seems to exist a systematic relationship between excess risk and share price. Since variations in risks can be measured and modelled, it is possible to keep portfolio risks in check and to help improving overall performance.

The recent Yale-University study of Moreira and Muir (2017) confirms this phenomenon. They compared the performance of U.S. stock portfolios pursuing two different investment strategies. One strategy adopted a typical “Buy & Hold” approach, the other followed a risk-based strategy that decreased exposure during high-volatility periods and increased it during low-risk periods. The authors found that over a period of 90 years (1926-2015) the risk-based investment approach delivers better risk-adjusted returns, on average an additional 2% annually, than “Buy & Hold”\textsuperscript{15}. Figure 11 depicts the logarithmic performance of both strategies.

The Yale study further expands the investigation by examining the indexes of 20 OECD countries. There, the authors found that, in 16 out of 20 cases, the risk-based approach achieved a higher Sharpe-Ratio\textsuperscript{16}. Over all 20 countries, the average improvement of the Sharpe-Ratio amounted to 0.15.

\textsuperscript{15} In the Yale study, a difference of 2% p.a. signifies that $1,000 invested in 1926 would be worth about $4,000,000 today, rather than $20,000,000.

\textsuperscript{16} The Sharpe ratio is a common measure to express risk-adjusted performance and is defined as the average return earned above the risk-free interest rate, divided by a portfolio’s standard deviation. The ratio expresses how effective risk-taking activities were per unit of additional risk assumed.
These findings demonstrate that effective risk management has the potential to deliver better risk-adjusted returns and, given that the investor’s risk “limit” allows for it, may also generate higher absolute returns compared to a “Buy & Hold” strategy.

### 4.5.3 Risk and Diversification

The study of Haas and Mittnik (2009) shows that positive excess risks do not only coincide with negative share prices, but are typically also accompanied by correlation surges. The authors investigated the common behaviour of three stock indexes, the S&P 500, FTSE 100 and the DAX, based on so-called “Markov Switching” multi-regime models. Their model assumes that every return observation originates, with a regime-specific probability, from a set of regimes which are characterised by an individual multivariate normal distribution.

Whilst a one-regime model conforms to the common multivariate normal distribution, parameterised by (historical) averages, volatilities and correlations, multi-regime models can approximate non-normalities with arbitrary precision. By letting the number of regimes grow, non-normalities such as asymmetry, fat-tails and non-linear dependencies between assets, which are commonly found in financial data, can be easily modeled. Although linear dependencies are postulated within each regime by using correlation as dependence measure, complex dependencies can be captured in terms of local, linear approximations specified by the different regimes.

The mean returns, volatilities and correlations of the indexes for the sample period 1991 - 2001 are shown in Table 3. The data included in Haas and Mittnik (2009) ranges from March 1991 to August 2005 of which the first ten years are used for estimations, while the remaining data is used for validation of the (out-of-sample) predictions.
4.5 The Interaction Between Risk, Return and Diversification


<table>
<thead>
<tr>
<th>Index</th>
<th>Annual return</th>
<th>Volatility</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>FTSE</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>13.05</td>
<td>14.11</td>
<td>0.54</td>
</tr>
<tr>
<td>FTSE</td>
<td>7.49</td>
<td>15.29</td>
<td>-</td>
</tr>
<tr>
<td>DAX</td>
<td>12.22</td>
<td>18.69</td>
<td>-</td>
</tr>
<tr>
<td>Average</td>
<td>10.92</td>
<td>16.03</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 4: Estimation results for multivariate multi-regime models for weekly returns on the S&P 500, FTSE 100 and the DAX. Returns and volatility are annualised. Reported are deviations of the average regime values from the conventionally estimated values given in Table 3. Sample period: March 1991–Feb. 2001. Source: Haas and Mittnik (2009)

<table>
<thead>
<tr>
<th>Number of Regimes</th>
<th>Volatility-Regime</th>
<th>Probability (%)</th>
<th>Φ Volatility-Deviation</th>
<th>Φ Return-Deviation</th>
<th>Φ Correlation Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>High</td>
<td>39.8</td>
<td>3.92</td>
<td>-2.83</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>60.2</td>
<td>-3.39</td>
<td>1.87</td>
<td>-0.17</td>
</tr>
<tr>
<td>3</td>
<td>High</td>
<td>42.5</td>
<td>3.64</td>
<td>-1.44</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>44.1</td>
<td>-3.01</td>
<td>1.13</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>Very Low</td>
<td>13.4</td>
<td>-5.96</td>
<td>8.75</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

namely 10.92% for annual returns, 16.03% for annualised volatilities and 0.55 for the correlations. The corresponding regime-specific estimates, derived in Haas and Mittnik (2009), are presented in Table 4.

In the two-regime case, a high-volatility regime with an increase in volatility of 3.92% leads to a drop in returns of 2.83%. The second regime, a low-risk regime, shows a 3.39% decrease in volatility and is thereby one-third below the average. This reduction in volatility coincides with an annual return increase of 1.87% – compared to the high-risk regime this is an increase in returns of more than half. In periods of high market stress, correlations are 0.10 above and in calmer periods 0.17 below the overall correlation. The discrepancy between correlations in high- and low-risk phases is substantial: 0.65 versus 0.38. With a probability of 60%, the three markets were mainly in calmer waters during the sample period.

As far as the three-regime model is concerned, a similar pattern arises, but with the low-volatility regime of the two component case being split further into moderate and a very low regime, with a volatility drop of -3.01% and -5.96%, respectively. The latter amounts to half of that of the the high-volatility regime. Whilst the returns in both of the higher-volatility components are below the overall average (-1.44% and -1.13%), they experience a substantial surge of 8.75% during low volatility. In addition, the spread of the correlations across the three regimes increases: In high-risk phases it amounts to 0.64, in the lowest-risk regime to only 0.29 - a difference of 0.35.

The results of Haas and Mittnik (2009) not only signify that positive (negative) excess risks tend to coincide with negative (positive) equity returns, they also highlight that, during high-volatility markets, (Markowitz-type) investment approaches with long-term correlation estimators systematically overestimate the beneficial efficacy of correlation-based diversification on risk reduction. In other words, when markets turn south, long-
term correlation-based diversification does not deliver what it promises, but time-varying correlations need to be taken into account.

4.6 The Allocation Decision

The allocation decision, which is generated by Scalable Capital’s automated investment process, reflects the risk-return tradeoff of each individual investor:

- maximise the reward associated with the expected portfolio return,
- penalise implicit costs that arise, for example, from excessive trading and large bid-ask-spreads,
- comply with the individual risk tolerance expressed in terms of a 95%-VaR-limit,
- penalise concentrated allocations within each asset class, and
- ensure that the weights of the instruments stay within their prescribed bounds.

Due to its dynamic nature, the allocation decision cannot be formulated as an isolated single-period optimisation problem. Dynamic risk modelling as well as transaction costs induce intertemporal dependencies between allocation decisions of specific periods. A rebalancing decision today for example automatically affects the starting allocation in future periods and hence, the transaction costs of a future rebalancing event.

In mathematical terms, Scalable Capital solves the following constrained optimisation problem in each period and for each investor $i_k$ in risk category $k$:

$$\max \mu((w_{ik,t})_t) - p((w_{ik,t})_t) - \Delta((w_{ik,t})_t), \quad i_k = 1, \ldots, I_k, \quad k = 1, \ldots, K, \quad (4)$$

subject to the restrictions

$$\text{VaR}((w_{ik,t})_t) \leq \text{VaR}_k^*, \quad (5)$$

$$lb_{gk} \leq a'_{gk} w_{gik,t} \leq ub_{gk}, \quad \text{for all instrument groups } g = 1, 2, \ldots, G, \quad (6)$$

$$1'w_{ik,t} = 1, \quad (7)$$

where:

- $\mu(\cdot)$ portfolio return
- $p(\cdot)$ expected transaction costs due to rebalancing
- $\Delta(\cdot)$ penalty for deviations from market-capitalization based weights
- $w_{ik,t}$ $N$-dimensional vector of weights for portfolio in $t$ of investor $i_k$
- $(w_{ik,t})_t$ series of $N$-dimensional weight vectors over future periods
- $\text{VaR}_k^*$ VaR-limit for risk category $k$
- $\text{VaR}(\cdot)$ projected portfolio VaR
- $lb_{gk}$ lower group-bound for instruments in group $g$ and risk category $k$
- $ub_{gk}$ upper group-bound for instruments in group $g$ and risk category $k$
- $a_{gk}$ asset-selection vector for group $g$ and risk category $k$

The attractiveness of conventional MVO lies in its mathematical simplicity. However, the optimisation problem (4)–(7) becomes much more complex and computationally demanding in the context of dynamic risk management: A MVO problem with $N$ unknowns becomes a high dimensional optimisation problem in the multi-period case for which global solutions cannot be computed in one step. Therefore, the problem must
be broken down into a multi-step optimisation problem. This approach allows to model all components of the optimisation problem such that they realistically approximate the empirical reality of financial markets. This is in contrast to a conventional MVO which makes strong simplifying assumptions that ensure the applicability of standard mathematical tools for solving this oversimplified problem.

By not relying on unrealistic assumptions, the solution to Scalable Capital’s intertemporal optimisation problem (4)–(7) turns out to be considerably less straightforward. Additional computational burden arises from the fact that portfolios are optimised on a client- rather than a risk-class-level. As a consequence, portfolios within a given risk category will be very similar but, in general, not identical. The differences in portfolio weights for two investors in the same risk category can have several reasons:

- a difference in magnitudes or the timing of in- and outflows,
- different portfolio sizes that will affect weights due to a lack of fractional investment, shares, and
- different tax situations.

It is for these individual differences that Scalable Capital prefers to optimise the portfolio of each client individually. This implies a substantial computational burden, but is clearly in the best interest of our clients.

4.7 Risk-based Allocation in Action

The previous section introduced the motivation for our risk-based investment process and expanded on its principles. In order to look at how the allocation mechanism works in reality, we observe how portfolio weights change based on backtests between the beginning of 2000 and the middle of 2018. For conducting the backtests over this extended period of time, we have chosen a representative investment universe for the current Scalable Capital investable assets - see table 5.

Figure 12 shows the price evolution of the different asset classes (top graph) and the asset class weights over time for two fixed risk categories: A low-risk category with a
4.7 Risk-based Allocation in Action

5% VaR target and higher-risk category having a 25% VaR target. The colors of the asset class weights correspond more or less to their riskiness: Dark blue represents less risky asset classes (such as money market), whereas increasing brightness of the colors indicates increasing riskiness of the underlying asset class.

Both charts showing asset class weights over time exhibit dynamically changing portfolio weights. With increasing volatility of a specific asset class, and hence with typically weakening performance, Scalable Capital’s portfolio management reduces the portfolio’s exposure to this asset class – if needed up to a full liquidation of the investment. In phases of weak stock markets, as for example after the Dot-com Bubble Burst in spring 2003, the Subprime Mortgage and Financial Crisis in 2007-2009 or the Chinese Stock Market Turbulences at the end of 2015 to beginning of 2016, equity exposure was reduced significantly in both risk categories. Low-risk asset classes, indicated by the course of the blue weights in Figure 12, predominate in these periods. When markets rebound, higher-risk allocations are quickly re-established. This can be seen by the increasing exposure to turquoise and yellow asset classes.
Figure 12: Price trajectories (scaled) of representative asset classes and exemplary risk-based asset allocations for portfolios with 5% and 25% VaR targets.
5 Regular Reviews and Updating Client Profiles

Investors’ financial statuses and personal situations are likely to change over time. Such changes, and the fact that investment goals or risk attitudes are not carved in stone, may require an update of the investor profile and, possibly, a realignment of the investment strategy. Regular reviews of client profiles are therefore part of the Scalable Capital investment process. Once an investor reports changes to his or her profile, Scalable Capital will examine to what extent investor-specific parameters need to be adapted. Scalable Capital will regularly inform their investors about their profile currently on record and ask them to verify and, if needed, to update their profile. Such updates will automatically trigger a re-evaluation of the appropriateness of the investor’s risk classification and may – after consultation with the client – lead to an adjustment of the investment strategy.

Regulation and investment practices are perpetually changing. Scalable Capital systematically monitors such changes and may respond by adapting its processes to be inline with new regulation or investment practices.
References


