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1 Overview
Scalable Capital offers intelligent and cost-efficient services for systematic wealth creation. Services that are commonly available only to large institutional investors or high-net-worth individuals. Scalable Capital’s investment process is built around the client, taking both investment goals and individual risk tolerance explicitly into account. Based on the investor’s individual profile and a carefully selected, comprehensive universe of index-based investment instruments, Scalable Capital deploys a highly innovative, rule-based algorithm to construct a tailor-made, optimal investment portfolio. All portfolios are continually monitored and rebalanced in order to keep them in line with the specific investment goals and risk limits.

1.1 The Building Blocks
The Scalable Capital investment process combines five building blocks that pave the way to optimal wealth creation:

▷ Compilation and ongoing adaptation of the relevant investment universe to always guarantee a comprehensive, globally diversified and cost-efficient set of investment opportunities.
▷ Determination of the investor profile in terms of investment goals, financial status and risk tolerance.
▷ Derivation of an investor-specific, optimal asset allocation.
▷ Continual process of monitoring and dynamically adjusting each individual portfolio in line with the investor’s risk tolerance.
▷ Regular review and updating of the investor profiles.

All these building blocks have been carefully designed and tuned to one another so that they form a comprehensive and effective wealth creation process.

1.2 The Process
The investor’s individual profile and the characteristics of the instruments in the investment universe are the central inputs for Scalable Capital’s investment process as summarised in Figure 1. Individual investment goals and risk tolerance specify the requirements the investment process has to comply with throughout the wealth creation. The current market environment, as described by the long- and short-term behaviour of the asset classes under consideration, provides the basis for statistically projecting future market behaviour and uncertainties.

Strongly believing that the creation of wealth is best accomplished by not losing any in the first place, Scalable Capital’s asset allocation algorithm focuses on downside-risk protection. Once the investor’s optimal portfolio is invested, Scalable Capital will continually monitor and project its development and assess whether it is on track or not. If the projection falls outside the prescribed risk corridor, the optimisation algorithm steps in automatically and re-optimises the portfolio.

This process differs substantially from commonplace Markowitz-type approaches. By using simple, backward-looking, volatility-based risk measures, these approaches fail to account for risk dynamics and the omnipresent asymmetry in the up- and downside potential of financial investments. As a consequence, Markowitz-type approaches tend to overestimate profit chances and underestimate loss potentials.

Rather than scheduling calendar-driven rebalancing (say, biannual or annual) inter-
vals, as is common with conventional investment strategies, Scalable Capital’s rebalancing activities are mainly driven by projected downside risk of the portfolio. In addition to risk concerns, there can be other rebalancing triggers, such as violations of portfolio weight limits, the in- or outflow of funds, or changes in the investor’s profile, which is re-assessed on a regular basis.

With its permanent risk monitoring and risk control, Scalable Capital provides investment services that are rarely afforded to private investors. Even in cases where services of this sort are offered, they typically come with rather traditional assumptions and procedures – such as naive risk parity approaches – and are based on simple, backward-looking asset-specific volatility or historical simulation instead of realistically calibrated, forward-looking Monte Carlo simulations as Scalable Capital does.

In the following sections we describe and explain the major building blocks of Scalable Capital’s investment process.
2 Investment Universe

Diversification is essential for designing successful risk-controlled investment strategies and achieving attractive risk-return profiles. Scalable Capital has, therefore, defined a broad and global universe across asset classes that cover the relevant security markets. This greatly helps to control risk through diversification, while letting the investor still benefit from the asset classes’ growth potential.

2.1 Diverse Asset Classes

Scalable Capital’s investment universe comprises all liquid asset classes that can be traded in a cost-efficient manner. It includes stocks, government and corporate bonds from all relevant developed and emerging economies, covered bonds, real estate stocks, natural resources, plus money market or cash-like instruments. The composition of the investment universe is continually monitored and adapted in order to pave the way for effective wealth creation.¹

2.2 ETFs: Cost-efficient, Passive Instruments

Rather than investing in individual instruments (i.e., stocks or bonds) belonging to an asset class, Scalable Capital invests in funds, i.e., baskets of investment instruments, which represent a specific asset class. This guarantees diversification within each asset class and, together with the already highly diversified universe, enhances overall diversification.

In order to do this at low cost, Scalable Capital selects passive index funds, so-called Exchange Traded Funds (ETFs), as investment vehicles. The only asset type where Scalable Capital may deviate from the strict adherence to ETF instruments is the cash/money market segment as other, low-cost alternatives may be available.

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**Exchange Traded Fund (ETF)**

An *Exchange Traded Fund* (ETF) is a fund that tracks an index of assets like stocks, bonds, commodities etc. Different from conventional funds, ETFs are directly traded on securities exchanges – just like stocks. Buying an ETF means buying shares of all assets that belong to the index. ETF managers do not try to outperform the index, they simply try to match its performance in a reliable and cost-efficient manner.

*Exchange Traded Funds* have a number of attractive advantages over conventional mutual funds:

**Passive Management:** An ETF tracks a specific market index and does not try to outperform it. Actively managed funds attempt to outperform a benchmark index and often invest significant resources into research in an attempt to do so. In the medium and long run, the vast majority of actively managed funds are not able to outperform their benchmarks. This is for two reasons: for one, it is hard to consistently “outguess” the market, and secondly the expenses for research (teams) are nontrivial and add substantially to a fund’s management cost. In contrast,

¹ The current composition of Scalable Capital’s investment universe including the specific investment instruments can be found on our website.
ETFs run on autopilot and require only minor adjustments to keep them in line with the index they track. Therefore, the expenses ETFs incur are much lower. The annual management expenses of ETFs can range from about 0.1% of the investment for mainstream indices and up to about 0.75% for highly specialised and difficult to replicate indices. Management fees for actively managed funds are typically in the range of 1.5% annually, but can be considerably higher when focusing on more exotic assets.

**Simplicity:** An investment in an ETF means investing in a basket of assets with a single transaction. There is no need to make a multitude of transactions for individual stocks or bonds which create excessive burden and multiply transaction costs. Also, since they are traded on exchanges, ETFs do not have initial fees (also called “sales fees”) or exit fees as mutual funds commonly do.

**Flexibility:** The price of mutual funds is typically set once during a day, fixing the price at which fund shares can bought or sold. Like publicly traded stocks, ETFs are traded throughout the exchange’s trading hours and can be sold and bought at prices that are continuously updated during these hours.

**Transparency:** Mutual funds reveal their specific investments only very infrequently, consequently, most of the time, it is not clear in which assets they are invested. ETFs usually track a well-defined index, whose constituents are commonly known or published by the index or ETF provider.

All in all, ETFs are cost-efficient, diversified, flexible, and easy-to-understand investment vehicles, making them ideal instruments for wealth creation. Typically, for any given asset class, there is a range of ETFs on the market, offered by competing providers. Scalable Capital applies a rigorous selection process to identify the ETFs that serve its clients best.

### 2.3 Scalable Capital’s ETF Selection Criteria

Scalable Capital applies a range of quantitative and qualitative criteria when deciding on which ETF to include or to exclude from its investment universe:

**Quantitative Criteria:**

- **How expensive?** The *Total Expense Ratio* (TER), i.e., the ratio of the fund’s total cost over its total assets, is a measure of a fund’s overall expenses. In the investor’s interest, Scalable Capital favours low-TER ETFs.

- **How liquid?** Illiquid ETFs, i.e., ETFs with low trading volume, have larger bid-ask spreads, which increase transaction costs. We focus on large (in terms of market capitalisation) and established (in terms of issuance date) ETFs which have multiple designated market makers and liquidity providers. Everything else equal, we prefer ETFs with high liquidity and low spreads.

- **How exact?** In other words, how well does the ETF track the underlying index? ETFs that follow the strategy of *full physical replication* try to invest in the very assets the index is composed of and match the index weights more or less exactly. The *representative sampling* strategy invests only in a representative subset of the index constituents. This is typically less expensive than full physical replication,
but reduces tracking precision. Scalable Capital prefers ETFs with low tracking errors over less accurate ones.

- **How diversified?** ETFs usually track broad market indices with dozens, sometimes even hundreds of different components. This ensures exposure to the fundamental factors driving returns in a specific asset class without taking excessive risk to idiosyncratic events such as bankruptcies or sovereign defaults. On the other hand, it becomes increasingly expensive to track very broad indices as they include a significant portion of less liquid components (the so called “long tail”). Scalable Capital’s selection process balances the need for sufficient diversification with the requirement of keeping TERs low.

**Qualitative Criteria:**

- **How risky?** Apart from the general market risk, induced by the ups and downs of its constituents, an ETF investment can face other types of risk. Rather than buying the underlying assets, some ETFs use synthetic replication to track an index by entering into swap or derivative agreements with third parties. In this case, an ETF is exposed to counterparty risk, because losses may arise if a counterparty does not fulfill its part of the deal. Other ETFs engage in securities lending which might boost investor returns and managers’ revenues but also introduces additional risks. Everything else equal, Scalable Capital favors ETFs that have, by construction and management practices, low risk profiles.

- **How tax-friendly?** The design of an ETF can have tax implications for the investor. As tax legislation varies from country to country, Scalable Capital takes transparency and simplicity with respect to taxation into account when determining or adapting a country’s ETF universe.

- **How convertible?** ETFs are being issued in many different currencies and listed on many different markets. Scalable Capital favors instruments which trade in the investor’s home currency to mitigate foreign currency conversion costs.

- **How fractional?** In some jurisdictions, ETFs can only be bought and sold in integer quantities. Therefore, the price of an ETF determines the minimum increment when rebalancing the portfolio. The smaller the price is, the more accurate the adjustment. Everything else equal, Scalable prefers ETFs with low absolute prices.

Typically, there is not one ETF in a given peer group that dominates the others with respect to all of the above criteria. Scalable Capital tries to find the best balance when selecting ETFs to represent an asset class. For each country in which Scalable Capital operates, the ETF selection will change over time as new ETFs are issued or the attributes of existing ones change. Scalable Capital’s website provides up-to-date information about the current set of instruments making up the country-specific investment.

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2 Such risks arise when a non-synthetic ETF lends out (typically only a small fraction of its) assets against a fee. To avert the counterparty risk, the borrower generally posts a cash collateral which will be liquidated if a borrower does not return the assets. Two loss scenarios can arise: first, although the collateral is regularly adjusted, at liquidation it may fall below the value of the assets lent out; second, the ETF company itself may invest the cash collateral in a money market fund that could lose money.
3 Risk Measurement and Risk Classification

Both investment goals and risk tolerance are crucial inputs for deriving a client’s appropriate asset allocation. While it is reasonable to assume that investors have no desire to limit profits on their investments, their willingness or ability to bear losses is clearly limited. With this in mind, Scalable Capital treats an investor’s risk tolerance as a constraint that the asset allocation has to satisfy at all times and derives the most attractive portfolio among all risk-compatible allocations.

This requires a sensible and reliable measure of investment risk and comprehension of this by the investor. Since investors associate losses rather than price fluctuations with risk, Scalable Capital focuses particularly on the use of downside-risk measures. There are several measures to choose from. Therefore, we will first introduce the different risk measures, discuss their advantages and disadvantages and their role in Scalable Capital’s investment process, before explaining the underlying risk classification scheme.

3.1 Risk Measurement

3.1.1 Alternative Concepts for Risk Measurement

Before assigning an investor a specific risk category, the question of how to measure investment risk arises. Conventional portfolio management approaches are based on mean-variance optimisation and use the statistical concept of standard deviation (also called volatility) to measure investment risk.

Standard deviation, volatility and variance

The standard deviation or volatility is a statistical measure of dispersion and is defined by the square root of the variance. The variance is defined as the expected squared deviation of the return on an investment from its expected return.

The use of standard deviation or volatility has, however, serious drawbacks:

▷ Although it is a commonly used statistical measure of dispersion, volatility is difficult to interpret, even for professional investors.

▷ Even if we know the volatility of an investment, it is not clear how to infer the actual loss potential from this information.

▷ Since it is based on squared deviations, volatility is a symmetric risk measure that averages loss and profit potentials. Therefore, to be a valid measure for loss potentials, gains and losses need to behave in a similar fashion. But it has been long established that in reality this is not the case. We commonly observe that extreme losses tend to be much larger than extreme gains.³ Example 1 in Section 4.1.1 highlights this behaviour for a wide range of asset classes.

In view of these deficits, Scalable Capital’s investment process relies on alternative and more appropriate risk measures. These measures capture, in one way or another, the loss potential or downside risk of a portfolio rather than simply its up- and downside variations. A commonly used measure for downside risk is called Value-at-Risk (VaR). It is widely used in the regulation of financial institutions and defined by the loss level

³ See, for example, Mandelbrot (1963), Westerfield (1977), Mittnik and Rachev (1993), and Kim, Lee, Mittnik and Park (2015).
3.1 Risk Measurement

that is not to be exceeded with some – typically small – probability. Another measure, that is closely related to VaR, is the Expected Shortfall (ES). ES looks at the expected loss when VaR is violated. Finally, the Maximum Drawdown (MDD) is a measure that focuses solely on the worst investment outcome.

**Value-at-Risk, Expected Shortfall and Maximum Drawdown**

*Value-at-Risk* (VaR) is a downside risk measure for an investment. For a given holding period, the VaR at the \( p\% \) confidence level indicates the potential loss at the end of the holding period that, with a probability of \( p\% \), is not to be exceeded. From a statistical viewpoint, the \( p\% \)-VaR corresponds to the \((100-p)\%\)-quantile of an investment’s return distribution. The *Expected Shortfall* (ES) builds upon VaR and is defined as the loss that is to be expected in those cases where the VaR limit is violated. The *Maximum Drawdown* (MDD) is given by the worst outcome that is to be expected.

Each of these risk measures exhibits advantages and disadvantages which are discussed below in order to show their appropriateness or inappropriateness for use in the investment process.

3.1.2 VaR, ES or MDD?

The suitability of a risk measure for managing a portfolio according to the client’s risk specification depends on various criteria. Important criteria are:

- Easy to comprehend for investors
- Suitable for the chosen investment instrument
- Suitable for backtesting and empirical model validation
- Robustness in terms of computation and prediction
- Computational complexity

The following discussion shows that all three risk measures VaR, ES and MDD perform differently when taking these criteria into account.

Considering *comprehension*, MDD comes off well, since “maximal loss” represents the largest possible loss for a given period of time which will never be breached. One can say that the MDD is a limit case of VaR as the confidence level approaches 100%. Hence, the VaR corresponds to the MDD concept but uses a 95% probability instead of a 100%. ES is a conditional expected value: the expected loss under the condition of exceeding a VaR-value with a given confidence level. The interpretation of a conditional expected value is not trivial especially when applied to events that occur with small probability, i.e. when operating with high confidence levels. In addition, it is hard for an investor to assess whether or not an observed loss is in line with a given ES-target.

Its *suitability for the asset class under consideration* is relevant when choosing a risk measure. An important criterion is whether or not it can handle high-risk assets in a portfolio. A high-risk investment is characterised by having a fat-tailed return distribution. Statistically, fat-tailedness can be measured in terms of the so-called tail-index. The lower the tail-index, the higher the possibility of extreme events. Although ES is attractive in the sense that it looks beyond VaR, it has the disadvantage that it does

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The value of ES can be computed using the following formula:

\[
ES = \frac{1}{1-\alpha} \int_{\alpha}^{1} V \left( \frac{r}{\alpha} \right) dr
\]

where \( V \) is the probability distribution function of returns and \( \alpha \) is the confidence level.

See, for example, the explanations on Wikipedia on [https://en.wikipedia.org/wiki/Conditional_expectation](https://en.wikipedia.org/wiki/Conditional_expectation).
3.1 Risk Measurement

not exist and, thus, cannot be calculated when an investment has a tail-index of one or below. Whereas VaR and MDD always exist regardless of the underlying distribution. In order to circumvent this ES deficiency, a common strategy is to truncate price jumps that exceed some – often arbitrarily – chosen level or to assume that returns beyond that level follow again a thin-tailed distribution. So-called “Truncated Levy Flight” or “Tempered Stable” models fall into that category. Clearly, such approaches are questionable as they assume that moderate price changes are fat-tailed whilst larger ones follow a normal or thin-tailed distribution.

In addition, the suitability of a risk measure may also depend on the types of instruments making up the investment universe. In the case of non-linear\(^5\) instruments (like Standard-, Barrier-, Knockout-, Lookback- etc.) options, a VaR-measure might not be sub-additive. Subadditivity is a theoretically interesting property when aggregating risks. There, subadditivity precludes a potential overestimation of the aggregated risk. In other words: Subadditivity prevents an exaggeration of portfolio risks that arises when being overly cautious in assessing risk reduction from diversification. ES and MDD are always subadditive regardless of the nature of the universe. However, subadditivity does not need to hold true even when dealing with linear investments. In cases where the tail-index is one or less, the aggregation will increase rather than reduce overall risk. In contrast to ES, VaR is capable of reproducing “superadditivity” (cf. Hyde, Kou und Peng, 2009; Ibragimov, 2009).\(^6\)

Since the investment universe of Scalable Capital consists only of ETFs and does not contain complex, non-linear investment vehicle, the possibility of inappropriate additivity behaviour when using VaR does not arise. This is also demonstrated in a study by Danielsson et al. (2005). In a linear investment universe, ES turns out to be redundant measure as it moves proportional with VaR. In other words, consistent use of VaR or ES will lead to equivalent allocation decisions when dealing with linear portfolios.

One of the most important criteria leads to the question to which extent the risk measure allows for backtesting and empirical model validation. The quality of a risk model as well as the effectiveness and robustness of an allocation model can, ex-ante, only be reliably assessed by the use of (retrogressive) backtests and empirical validation. A severe disadvantage of ES, compared to VaR, is that it lacks so-called “elicitation” making proper validation practically impossible. This implies that an investor has to “blindly” trust the risk model and learns about its adequacy afterwards. According to Davies (2013): “... verifying the validity of mean-based [ES] estimates is essentially more problematic than the same problem for quantile-based [VaR] statistics”\(^7\). To handle

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\(^5\) A financial instrument is linear, when the returns are linearly dependent on the underlying risk driver, as it is the case for stocks, for example.

\(^6\) Dhaene, Goovaerts and Kaas (2003) as well as Heyde, Kou and Peng (2009) and others doubt the reasonableness of subadditivity for risk measures. The following example, adapted from Dhaene, Goovaerts and Kaas (2003), outlines this process. In the case of risk disaggregation, subadditivity implies that when risks are disaggregated, the sum of partial risks can not be smaller compared to the overall risk before the split. Let us assume a bank in financial distress with a total loss incoming. The banking supervision authority arranges a division into a “sickish bad bank” and a “healthy good bank”. If the total loss still threatens the bad bank but not the good bank anymore, the sum of partial risks is smaller than the risk before the division. A subadditive risk measure is unable to model this superadditive situation.

\(^7\) Due to this fact, Paul Embrechts, a leading financial risk-researcher, comes to the conclusion: “If you held a gun to my head and said: ’We have to decide by the end of the day if Basel 3.5 should move to ES, with everything we know now, or do we stick with VaR’, I would say: ’Stick with VaR’” (Risk.net, March 2013, http://www.risk.net/risk-magazine/news/2253463/mooted-var-substitute-cannot-be-backtested-says-top-quant).
3.1 Risk Measurement

these difficulties with model validation, Emmer, Kratz and Tasche (2015) propose to approximate backtesting of ES-based models by deriving VaR estimates for different confidence levels. Here, the question arises whether VaR-based methods shouldn’t be the preferred choice in the first place.

Only forward-looking risk management can be effective. Therefore it is important that risk measures are *robust with respect to computation and prediction*. Robustness is especially problematic for risk measures like ES and MDD since they need extreme-tail information. By definition, tails describe the far ends of a distribution. When considering losses, the tail is given by the single most extreme loss observed. Since statistical analyses can not be conducted with a single observation, one needs to “improvise”. Two strategies can be adopted to avoid this problem: Looking at more losses than just the most extreme loss, or postulating assumptions about the distributional behaviour in the tail area. The problem with the first strategy is that, although risk estimates can be stabilised, one tends to lose information about the actual distributional properties in the tails. The more non-tail observations are considered the less one learns about the tails and the more risk-measure estimates become distorted. On the other hand, using only a few, extreme observations induces less robustness because estimates become rather erratic. The second strategy suffers from high sensitivity with respect to the distributional assumption for the tail. It is the belief of the modeller, and not actual data that predominantly determine risk assessment. The drawbacks of both strategies speak against the usage of ES and MDD as risk measures: Stable ES and MDD assessments tend to either rely on biased, data-based estimation procedures or on restrictive and often highly unrealistic assumptions like thin-tails and temporal independence of returns. Hence, both risk measures are not well suited for forward-looking risk and portfolio management. This conclusion is supported by Davis (2013): “… significant conditions must be imposed to secure consistency of mean-type [ES] estimates, in contrast to the situation for quantile [VaR] estimates ... where almost no conditions are imposed”.

Heyde, Kou and Peng (2009) suggest that the “Tail Conditional Median” (TCM) is a more resilient risk measure that can look beyond VaR. It corresponds to the ES but uses the conditional median instead of the conditional mean and can also be referred to as “median shortfall”. The median is more less sensitive to outliers compared to the mean. However, Heyde, Kou and Peng (2009) emphasise that for for linear investments TCM and VaR are equivalent. TCM at confidence level of 100-α% amounts to VaR of 100-\(\frac{\alpha}{2}\)%. If you are looking for a robust risk measure for a linear portfolio that reflects tail information and is robust, you should use VaR with an appropriately chosen confidence level.

In practical applications, *computational complexity* is desirable, since it simplifies the numerical derivation of optimal portfolios and helps to avoid suboptimal solutions. The difficulty of the MDD is its extreme sensitivity, which upstages numerical aspects and thereby prevents reliable portfolio optimisation\(^8\). Since VaR isn’t necessarily a convex function of the portfolio weights, VaR-based portfolio optimisation can be more burdensome compared to ES. Therefore, for computational reasons, ES is often preferred to VaR. However, modern computational procedures now allow the calculation of robust solutions for non-convex optimisation problems. For most applications, disregarding perhaps high-frequency trading or very large portfolios, the additional time and

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\(^8\) Chekhlov, Uryasev and Zabarankin (2005) state: “… however attractive the MaxDD measure is, the solutions produced using this measure in portfolio optimisation may have a significant statistical error because the decision is based on a single observation of the maximal loss.”.
3.2 Risk Classification

Table 1: Criteria for the suitability of risk measures

<table>
<thead>
<tr>
<th></th>
<th>VaR</th>
<th>ES (CVaR)</th>
<th>MDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple traceability</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Suitable for high risk investments</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Suitable for linear/ non-linear investments</td>
<td>Yes/No</td>
<td>Yes/Yes</td>
<td>Yes/Yes</td>
</tr>
<tr>
<td>Suitable for backtesting and model validation</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Robust calculability and predictability</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Computational simplicity</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

computational effort required for VaR-based optimisation may be well worth it.

Table 1 summarises the above discussion on the suitability of risk measures and reveals that no single measure dominates with respect to all criteria - each measure has certain advantages and disadvantages. As explained next, even though the overall advantages of VaR clearly prevail, Scalable Capital’s investment process makes use of all three risk measures.

3.1.3 Scalable Capital’s Selection

Since the different risk measures, VaR, ES and MDD, complement each other regarding their explanatory content, all three enter Scalable Capital’s investment process. However, given the differences in comprehensibleness for investors and the usefulness for backward- and forward-looking analyses, these risk measures are used in different stages throughout the investment process. All three measures, as well as other indicators, play a role when it comes to backward-looking analyses and ongoing monitoring processes. Because MDD is not suited for forward-looking analyses, it plays no role in controlling portfolios based on risk projections.

In view of the fact that, given a linear investment universe and proper usage, VaR and ES lead to equivalent asset allocations. Nonetheless, VaR is easier to understand than ES and hence plays the key part when it comes to the risk classification of investors. Once an investor’s appropriate VaR level has been chosen, Scalable Capital optimises, monitors and controls the portfolio with the aim stay close to that pre-specified VaR target.

The VaR is defined for the confidence level of 95% and an investment horizon of one year. Therefore, it can be viewed as the percentage loss over the next year that should not be exceeded with a probability of 5% or, in other words, in one out of 20 years the portfolio is expected to lose more than the specified VaR target.

3.2 Risk Classification

The initial step in Scalable Capital’s wealth creation process is to get to know the investor. This is not only a legal requirement, but also essential for deriving an investment strategy that is optimal for the client. The investor profile is obtained online, through a sequence of straightforward questions. The information is processed in real-time so that the investor learns right away about his or her risk classification and the corresponding asset allocation.
3.2 Risk Classification

Figure 2: Examples of typical profit-loss ranges for selected risk classes

3.2.1 Status and Investment Goals
The online questionnaire begins with questions about the investor’s personal and financial situation. Investment horizon, previous investment experience, and the understanding of specific investment instruments and their risk-return characteristics are queried. Next, risk-return preferences will be elicited based on the investor’s tolerance for loss and their goals for the investment. A consistency check runs in the background to assess whether or not Scalable Capital’s investment services are suitable for this particular investor. If this is not the case, the applicant will be informed about this and the onboarding process will be stopped.

3.2.2 VaR-based Risk Classification
Based on the answers to the questionnaire, Scalable Capital assesses for each investor his or her individual risk tolerance and determines the corresponding maximum VaR-limit. This limit reflects the investor’s risk capacity and risk attitude. Scalable Capital offers altogether 23 risk categories, ranging from 3 to 25, associated with the annual 95%-VaR-limits 3% to 25%. For example, an investor choosing risk category 12 wants to limit downside risk so that an annual decline of more than 12% should occur on average only once in 20 years.

When going through this elicitation process, the investor is assisted with graphical displays of potential profit-loss ranges (as depicted in Figure 2), which reflect the range of likely outcomes of investments associated with different risk classes (i.e., VaR-limits). The set of likely scenarios associated with a risk class is summarised in terms of the
expected median scenario and the central 90% confidence band. All these scenario summaries are exemplary depictions, as they are based on historically observed risk-return relationships.

Having completed the survey, Scalable Capital proposes the appropriate risk category. If the investor is not comfortable with this limit, he or she can choose a lower, but not a higher, risk limit. After the risk category has been selected, the portfolio composition that corresponds to that risk class will be shown.

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9 The bandwidths are defined by the 95%- and the 5%-quantiles of the scenarios.
4 Portfolio Optimisation

Given a carefully constructed investment universe as well as the individual risk classification, the question of how to derive a portfolio that best serves the investor leads to the central building block of Scalable Capital’s investment process. The objective is to generate the best long-term return without violating the investor’s risk limits.

What does best “serves” mean? Clearly, the typical investor prefers high-yielding over low-yielding investments. Unfortunately, investments with high return prospects tend to also have a higher risk of generating losses. This risk-return predicament forces the investor to find the “right balance” between risk and return prospects. A portfolio or asset allocation is deemed optimal, if its expected return is the highest of all permissible allocations that do not exceed the investor’s risk limit. This is essentially the optimisation problem the Scalable Capital investment process solves. Essentially, three ingredients enter Scalable Capital’s optimisation:

▷ The investor’s individual profile, reflecting investment goals and risk tolerance,
▷ the current market conditions, i.e., the risk-return prospects of the assets in the investment universe as well as their potential for risk diversification, and
▷ the costs from transacting and holding investment vehicles.

Based on this information, Scalable Capital’s optimisation engine, sketched in Figure 3, derives for each investor the individually optimal asset allocation.

Since avoiding large losses is crucial for successful wealth creation, Scalable Capital’s overriding concern is to control downside risk. To accomplish this, the portfolio weights are set such that the portfolio’s projected risk is compatible with the investor’s risk tolerance (VaR-limit). The reasons why downside-risk protection is instrumental for successful wealth creation and how Scalable Capital implements this protection will be explained next.

4.1 Downside Risk

Most quantitative portfolio managers rely on conventional mean-variance optimisation (MVO) for portfolio construction, developed by Harry Markowitz in the 1950s (see
Markowitz, 1952). At that time, MVO represented a major scientific breakthrough and has now become a cornerstone of modern finance. For his pioneering work, Markowitz was honoured with the Alfred Nobel Memorial Prize in 1990 and the Swedish Academy of Science wrote:  

Markowitz showed that under certain given conditions, an investor’s portfolio choice can be reduced to balancing two dimensions, i.e., the expected return on the portfolio and its variance. Due to the possibility of reducing risk through diversification, the risk of the portfolio, measured as its variance, will depend not only on the individual variances of the return on different assets, but also on the pairwise covariances of all assets.

Undoubtedly, Markowitz’ contribution represented a major breakthrough at the time, and the Nobel Memorial Prize was well deserved. There is, however, a crucial caveat in the statement of the Swedish Academy: The approach requires certain given conditions. Numerous empirical studies, starting with the pioneering works of Benoit Mandelbrot and Eugene Fama in the 1960s (see Mandelbrot, 1963; and Fama, 1965), have shown that essential assumptions underlying the Markowitz approach are critically violated in practice. As a result, MVO has fundamental practical shortcomings, which lead to inadequate asset allocations and flawed risk assessment.

### Covariance and correlation

Both covariance and correlation try to measure the dependence or “synchronicity” of the behaviour of two assets. The covariance looks at the simultaneous behaviour of the two assets’ returns in terms of deviations from their respective means. The expected value of the product of these two deviations defines the covariance. The correlation is a standardised version of the covariance (assuming values between +1 and −1) that is obtained by dividing the covariance by the two return volatilities.

A positive correlation indicates that both assets tend to jointly out- or under-perform their mean; a negative correlation indicates that the assets tend to move in opposite directions from their respective means. These tendencies become stronger as the correlation approaches ±1.

By using variance or volatility as a risk measure and covariance or correlation as a measure of dependence (i.e., as a measure of “synchronicity” in the co-movements of asset price), MVO implicitly assumes asset returns to have certain properties:

- Assets behave symmetrically in the sense that they have identical up- and downside potentials.
- The synchronicity with which asset prices move is constant regardless of whether we are, say, in a bear or bull market.

As extensive academic research and practical experience have shown – and as will be detailed next – these assumptions are soundly rejected by the vast majority of financial return data.

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11 See, for example, Mittnik, Paolella and Rachev, 2000; Rachev and Mittnik, 2000; and references therein.
### 4.1 Downside Risk

Table 2: 5%- and 95%-quantiles computed from monthly percentage returns of representative asset classes. Source: Center for Quantitative Risk Analysis

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Index</th>
<th>Label</th>
<th>5%-Quantile</th>
<th>95%-Quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>MSCI World</td>
<td>World</td>
<td>-12.2%</td>
<td>6.4%</td>
</tr>
<tr>
<td></td>
<td>DAX</td>
<td>DAX</td>
<td>-12.4%</td>
<td>7.9%</td>
</tr>
<tr>
<td>MSCI Brazil</td>
<td>Brazil</td>
<td>Brazil</td>
<td>-27.7%</td>
<td>14.4%</td>
</tr>
<tr>
<td>MSCI Russia</td>
<td>Russia</td>
<td>Russia</td>
<td>-29.2%</td>
<td>13.6%</td>
</tr>
<tr>
<td>MSCI India</td>
<td>India</td>
<td>India</td>
<td>-23.9%</td>
<td>13.7%</td>
</tr>
<tr>
<td>MSCI China</td>
<td>China</td>
<td>China</td>
<td>-19.0%</td>
<td>17.6%</td>
</tr>
<tr>
<td>Natural Resources</td>
<td>DJ-UBSCI</td>
<td>NatRet</td>
<td>-16.6%</td>
<td>8.7%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>REITs</td>
<td>REITs</td>
<td>-17.2%</td>
<td>9.8%</td>
</tr>
<tr>
<td>German Govt. Bonds</td>
<td>REXP</td>
<td>REXP</td>
<td>-2.1%</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

#### 4.1.1 Risk Measure

MVO uses volatility (i.e., the standard deviation of asset returns) to measure investment risk. Volatility is a symmetric risk measure that assumes an investment has identical loss and profit potentials. In other words, return distributions are supposed to follow a symmetric, bell-shaped curve, such as the well-known normal distribution. MVO ignores the widely-observed fact that large losses tend to be more pronounced and more frequent than large gains.

**Example 1** (Asymmetries in asset returns) Let’s take a look at the asset classes listed in Table 2. Computing monthly percentage returns over a 10-year period, the 5%- and 95%-quantiles (the 5%-quantiles correspond to 95%-VaR-levels), we find that the returns’ behaviour is far from symmetric. For most of the stock indices, large losses are almost twice as big as large gains. A similar finding holds for natural resources and real estate – an exception is the government bond index.

Findings of the type reported in Table 2 turn out to be the rule rather than the exception with financial return data. Therefore, symmetric risk measures provide a poor description of the true behaviour of financial markets and inevitably undermine efforts towards realistic risk assessment. Volatility as a risk measure

- Simply “averages” upside and downside potentials and, thus, tends to systematically underestimate loss potentials and overestimate upside chances,
- Does not provide clear-cut, reliable information about true loss probabilities and, as a consequence,
- Produces suboptimal asset allocations.

For these reasons, Scalable Capital’s portfolio optimisation process relies on an asymmetric downside-risk measure: the Value-at-Risk (VaR) measure introduced in Section 3.1. In accordance with the acceptable tolerance level, each investor is placed in a suitable risk category, which is associated with a specific 95%-VaR-limit over a one-year horizon. In other words, VaR-limit violations should, on average, occur only once in twenty years. The Scalable Capital algorithm treats this VaR-limit as a “hard constraint” that the projected risk of a client’s portfolio needs to satisfy at all times.
4.1 Downside Risk

Pearson

World
DAX
Brazil
Russia
India
China
NatRes
REITs
REXP

−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Tail Correlations

World
DAX
Brazil
Russia
India
China
NatRes
REITs
REXP

−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 4: Heat maps of conventional Pearson correlation estimates (left) and realistic tail-correlation estimates (right) from monthly returns on selected asset classes. The lower-left triangle below the diagonal (upper-right above the diagonal) in the tail-correlation matrix shows the correlations in the loss (profit) tail of the joint return distributions. (Source: Center for Quantitative Risk Analysis)

4.1.2 Measuring Dependence or Asset Co-Movements

Dependence measures try to capture the synchronicity of asset price co-movements. MVO measures dependence in terms of correlations – to be precise: in terms of (conventional) Pearson correlations. This amounts to assuming a linear relationship between assets and implies that asset prices tend to always move in a proportional fashion. It turns out that this can only be the case when assets follow a joint normal distribution. In other words: Only when price movements are normally distributed, correlation is a valid measure of dependence. The widely observed fact that asset co-movements are tighter in market downswings than in upswings clearly contradicts the normality or linearity assumption and makes the use of conventional correlation in asset allocation decisions more than questionable.

Example 2 (Asymmetries in correlations) Figure 4 depicts the correlation between the monthly returns of the asset classes listed in Table 2 in the form of so-called heat maps. Heat maps use colours to indicate correlation levels. High positive correlations (i.e., strong synchronicity) are shown in dark red and high negative correlations (i.e., strong contrary asynchronicity) in dark blue. The left matrix represents conventional or Pearson correlation estimates. The matrix on the right shows estimated tail correlations, where the triangle on the lower left below the diagonal reports the tail correlations in a down market (so-called left-tail correlations) compatible with the 95%-VaR measure, and the upper, right triangle the corresponding upside (or right-tail) correlations.

Since the conventional correlation measure does not distinguish between left- and right-tail correlations, MVO correlation matrices are always symmetric. The tail-correlation matrix reveals a high degree of correlational asymmetry, instead. The lower triangle is more “dark reddish” than is implied by the conventional correlation measure. Moreover, the upper triangle in the tail correlation matrix signals that overall synchronicity is lower during bull markets. For example, based on common correlation estimates, the asset class Natural Resources produces substantial diversification benefits (exceptions are – not surprisingly – the MSCI Brazil and MSCI Russia). The REXP also behaves untypically and turns out to be especially useful when seeking downside risk protection. Conventional correlation underestimates the REXP’s diversification benefits in bear markets.

Scalable Capital
4.1 Downside Risk

and generally overestimates synchronicity in bull markets.

This example illustrates that conventional correlation estimates obtained from financial return data tend to lie somewhere between left- and right-tail correlations. As the following example shows, by ignoring this phenomenon, MVO can overestimate diversification benefits in bear markets and is likely to fail when risk protection is needed most. Since it will also overestimate a portfolio’s upside potential during bull markets, MVO is bound to harm investors in both market regimes.

Example 3 (Asymmetric correlations and portfolio tail risk) The phenomenon that asymmetric correlations can have severe impact on the tail risk of portfolios is illustrated in Figure 5. It plots the left- and right-tail risk of an equal-weight portfolio of the stocks belonging to the Dow Jones Industrial Average. The left graph shows the downside (or long) tail risk associated with various quantile levels. The right graph illustrates the same for the upside (or short) tail risk. Shown are the empirical portfolio tail risks (black line) and fitted values derived from alternative dependence measures: conventional

\[ \text{Specifically, Expected Shortfall-related portfolio tail risk is shown. See Section 3.1.1 for and explanation of Expected Shortfall.} \]
4.1 Downside Risk

Pearson correlations (red), excess correlations (green), the tail correlation measure most commonly referred to in the financial literature, and, finally, Scalable Capital’s approach to assessing tail synchronicity (blue).

The graph shows that the portfolio’s true left- and right-tail behaviour greatly differ and that the potential for extreme losses can be more than twice as much as for extreme gains. Moreover, it reveals that both Pearson and excess correlations fail to adequately capture tail risks.

As the example succinctly demonstrates, conventional approaches, such as Pearson correlations or excess correlations, tend to be overly optimistic when assessing the potential for reducing downside-risk through diversification. Proper tail correlation measures, on the other hand, capture empirically observed tail risks much more realistically, provide more reliable downside-risk protection and, therefore, help to design reliable risk-controlled investment strategies without unnecessarily foregoing returns.

4.1.3 Temporal Dependence and Risk Clustering

Traditionally, asset managers see their main job as foreseeing future price movements of asset classes and individual assets, and they spend substantial resources on “research” to predict price trends. To do so, they resort to a range of different recipes using, for example, technical, fundamental, contrarian and various other philosophies. Clearly, being able to always accurately predict asset prices guarantees outperformance of the market. Furthermore, with accurate price predictions all market risk would be eliminated, and an investor could make unlimited, risk-free profits.

At Scalable Capital we do not believe in such paradise-like conditions and follow the overwhelming consensus from academic research, which suggests that financial markets are largely price-efficient. In other words: We take the view that the potential for outright, systematic price predictability for liquid assets is very limited.

Using a very long history of the Dow Jones Industrial Average the following analysis underpins this view. It also demonstrates that the prediction of market risk is much more promising than trying to directly predict price movements.

Example 4 (Return and risk predictability) Let us look at the Dow Jones Industrial Average (DJIA) index, one of the longest financial return series available, to examine return and risk predictability. Figure 6 shows the weekly DJIA index from 1896 to 2015. If future returns were predictable from historical returns, there should be temporal dependence between successive return observations. A common tool to assess the presence of temporal dependence in time series data is the so-called autocorrelation function (ACF). The ACF is a form of “memory function” that measures the strength and duration of (Pearson) correlation between increasingly distant return observations and, thus, is a measure of (linear) predictability.

The top panel in Figure 7 plots the weekly returns of the DJIA (left) and their ACF for a time horizon of up to one year or 250 trading days (right). The ACF shows that there is practically no linear predictability in the return series. All autocorrelations are near zero and mostly statistically insignificant (i.e., inside the 95%-confidence band indicated by the two horizontal red lines). From this we can conclude that past returns provide little or no information about future return behaviour.

To further analyse the question of predictability, let us decompose the return series into two components: the direction of the returns, expressed by a sequence of +1-values

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4.1 Downside Risk

Figure 6: Dow Jones Industrial Average index, weekly data from 1896 to 2015.

(or, in case of no change, 0); and the size of the return, i.e., the absolute value of the return. Expressed differently, an observed return can be written as the product

$$\text{Return} = \text{Return Direction} \times \text{Return Size}$$

or, mathematically, as $r_t = \text{sgn}(r_t) \times |r_t|$. The time series of the directional component simply tells us whether the DJIA has moved up or down on a particular day; and the size component essentially reflects the risk of an investment in the index: A large range indicates a large loss (or gain) potential.

If we can (linearly) predict the direction of the DJIA, autocorrelations of the directional component should be large and statistically significant. The center panel in Figure 7 shows the time series of past DJIA directions (left) and the corresponding ACF (right). It turns out that the directional ACF behaves very much like that of the returns: All autocorrelations are near zero and mostly insignificant. Thus, past directional behaviour does not tell us much about future up or down movements. This stands in stark contrast to the behaviour of the risk component, shown in the bottom panel of Figure 7: Autocorrelations are large and statistically significant over a horizon of more than a year (250 trading days), indicating that there is substantial predictability in risk.

The fact that all autocorrelations are positive implies that there is risk clustering in the DJIA: Periods of high (low) volatility tend to be followed by periods of high (low) volatility.

Example 4 illustrates that it is very hard if not impossible to come up with reliable asset-price predictions. The problem lies in the prediction of the direction of future price movements. Forecasting financial market risk, on the other hand, is much more promising given the strong empirical evidence for (linear) predictability. The econometrician Robert F. Engle received the Nobel Memorial Prize for developing a concept for predicting financial-market risk (see Engle, 1982). Scalable Capital exploits the fact that there is more predictability in market risk than in market direction and uses advanced methods that build on Engle’s original work.
4.1 Downside Risk

Figure 7: Linear predictability of returns on the DJIA index (top panel) and its components, namely, the return direction (center panel) and the return size (bottom panel). The data is shown on the left and the corresponding “memory functions” on the right, where bars ending inside the two red lines are statistically insignificant (noise) and those ending outside indicate significant linear predictability. The empirical memory functions suggest that neither returns themselves (top, right) nor return directions (center, right) can be predicted. There is, however, substantial predictability in risk (bottom, right).

4.1.4 Risk Projections

Scalable Capital uses methods from financial econometrics to generate risk projections. Since it cannot be expected that a single approach will dominate all others in all market situations, Scalable Capital employs a range of alternative strategies for VaR-predictions. They are all based on Monte Carlo simulations, in which we generate up to \( S = 10,000 \) or more scenarios for possible future asset price movements.

Scalable Capital’s Monte Carlo approaches have in common that they draw the scenarios from a model of the form

\[
    r_{t+1} = \mu_t(r_t, r_{t-1}, r_{t-2}, \ldots) + \varepsilon_t(r_t, r_{t-1}, r_{t-2}, \ldots),
\]

where \( r_t = (r_{1t}, r_{2t}, \ldots, r_{Nt})' \) denotes the vector of returns in period \( t \) of the \( N \) assets in the investment universe; and \( \mu_t(\cdot) \) and \( \varepsilon_t(\cdot) \) are the location function and random dispersion-from-location function, respectively, both of which depend on current and past market conditions. Let \( \hat{\mu}_t(\cdot) \) and \( \hat{\varepsilon}_t(\cdot) \) denote the empirically estimated versions of

\[1\] See, for example, Mittnik and Paolella (2003) and Kuester, Mittnik and Paolella (2006) for alternative, univariate projection strategies.
4.1 Downside Risk

these functions, then, scenario \( s = 1, 2, \ldots, S \), originating in period \( T \), can be derived recursively via

\[
\begin{align*}
\hat{r}_{s,T+1} &= \hat{\mu}_T(r_T, r_{T-1}, \ldots) + \hat{\varepsilon}_{s,T}(r_T, r_{T-1}, \ldots) \\
\hat{r}_{s,T+2} &= \hat{\mu}_T(\hat{r}_{s,T+1}, r_T, \ldots) + \hat{\varepsilon}_{s,T}(\hat{r}_{s,T+1}, r_T, \ldots) \\
\hat{r}_{s,T+3} &= \hat{\mu}_T(\hat{r}_{s,T+2}, \hat{r}_{s,T+1}, \ldots) + \hat{\varepsilon}_{s,T}(\hat{r}_{s,T+2}, \hat{r}_{s,T+1}, \ldots) \\
&\vdots \\
\hat{r}_{s,T+H} &= \hat{\mu}_T(\hat{r}_{s,T+H-1}, \hat{r}_{s,T+H-2}, \ldots) + \hat{\varepsilon}_{s,T}(\hat{r}_{s,T+H-1}, \hat{r}_{s,T+H-2}, \ldots),
\end{align*}
\]

where \( H \) denotes the projection horizon, i.e., the number of future periods to be projected. Applying this recursion for \( S \) replications, we can assess the range of possible outcomes and, in particular, obtain the one-year VaR-projection at the 95% confidence level, given by the 5%-quantile of the \( S \) one-year scenarios.

To illustrate this, let us look at the S&P 500 net total return index.\(^{15}\) The graphs in the top panel in Figure 8 plot the index and the daily returns from 04.01.1999 to 26.09.2014, where the index finished at 2,596 points. Employing recursion (2) to generate a one-year VaR projection with the information given on 26.09.2014, we obtain the scenarios shown on the left in the center panel in Figure 8. The histogram in the center right gives the distribution of the one-year scenarios and indicates the position of the 5%-quantile (i.e., the negative, one-year 95%-VaR), which amounts to a VaR of about 27%. The graph on the bottom left in Figure 8 shows all intermediate projections for the 5%- (red line) and 95%-quantiles (blue line). 90% of all scenarios lie within the cone formed by these two lines. Finally, the graph on the bottom right indicates the implied 90%-confidence band for the S&P 500 index level originating on 26.09.2014. According to the one-year projection, the index is expected to lie, with a probability of 95%, above 1,905 points.

VaR-projections can vary greatly as market conditions change. Figure 9 shows the 90%-confidence bands for the S&P 500 return (left graphs) and the level (right graphs) generated at four arbitrary dates. The one-year VaR-projection on 18.12.2000, shortly after the dot-com bubble began to deflate, was 34%. Towards the end of the U.S. 2007-2009 bear market, on 13.01.2009, the VaR-projection reached a very high level of 53%. The range of the corresponding confidence band, i.e., the second cone on the left in Figure 9 reflects heightening market uncertainty around that date. Shortly after that date, the S&P 500 rose significantly up to the end of 2014. In that growth period the projected market risk dropped sharply. On 24.03.2011, the VaR-projection fell to 28.5%, and on 26.09.2014 even further to 21%. This pattern is in line with empirical evidence that high VaRs tend to coincide with falling markets and low VaRs are often associated with rising markets.

The broad variation in riskiness also applies to other asset classes. Table 3 reports VaR-projections for alternative asset classes (including bonds, stocks, real estate, and commodities) for the four very same dates considered in the preceding S&P 500 illustration. Not all asset classes move in sync. Bonds vary much less than the other asset classes – not only in absolute but also in relative terms. For most asset classes, market risk is the highest on 13.01.2009. But there are exceptions: European Covered Bonds and Japanese Stocks peak on 18.12.2000 and 24.03.2011, respectively. These temporal variations in risk profiles clearly indicate that controlling risk by investing across dif-

\(^{15}\) Bloomberg ticker: SPTR500N Index. In the following we simply refer to the S&P 500 index.
4.1 Downside Risk

Figure 8: Illustration of Scalable Capital’s VaR projection strategy. The top panel plots the index and the daily returns from 04.01.1999 to 26.09.2014. A set of possible one-year scenarios is shown in the center left. The histogram in the center right reflects the distribution of these one-year scenarios and indicates the corresponding 95%-VaR (i.e., negative 5%-quantile). The bottom left graph shows all intermediate projections for the 5%- (red line) and 95%-quantiles (blue line), forming an envelope for 90% of the scenarios. The graph on the bottom right depicts the implied 90%-confidence band for the S&P 500 projections for up to one year.
4.1 Downside Risk

Figure 9: Relative (left) and absolute (right) S&P 500 90%-confidence bands at different dates. The lower branches of the bands correspond to 95%-VaR projections.
4.2 The Interaction Between Risk, Returns and Diversification

Table 3: 95%-VaR projections for various asset classes projected at different points in time. (Bold face numbers indicate the maximum VaR-projection for each asset class.)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Eur Covd Bonds</td>
<td>4.98</td>
<td>5.17</td>
<td>4.49</td>
<td>3.95</td>
</tr>
<tr>
<td>Eur Gov Bonds</td>
<td>5.64</td>
<td>6.82</td>
<td>5.88</td>
<td>5.96</td>
</tr>
<tr>
<td>Eur Corp Bonds</td>
<td><strong>5.89</strong></td>
<td>5.73</td>
<td>4.84</td>
<td>4.45</td>
</tr>
<tr>
<td>Eur Stocks</td>
<td>26.31</td>
<td><strong>40.01</strong></td>
<td>28.96</td>
<td>25.18</td>
</tr>
<tr>
<td>U.S. Stocks</td>
<td>34.05</td>
<td><strong>52.68</strong></td>
<td>28.54</td>
<td>26.99</td>
</tr>
<tr>
<td>Japan Stocks</td>
<td>36.34</td>
<td>38.82</td>
<td><strong>43.00</strong></td>
<td>32.27</td>
</tr>
<tr>
<td>Asia Pac xJap Stocks</td>
<td>31.18</td>
<td><strong>37.28</strong></td>
<td>31.06</td>
<td>26.31</td>
</tr>
<tr>
<td>Real Estate</td>
<td>21.15</td>
<td><strong>41.61</strong></td>
<td>23.75</td>
<td>22.49</td>
</tr>
<tr>
<td>Commodities</td>
<td>26.19</td>
<td><strong>36.55</strong></td>
<td>25.95</td>
<td><strong>18.61</strong></td>
</tr>
</tbody>
</table>

Different asset classes and employing a dynamic, risk-based asset allocation strategy is an effective and practicably implementable investment approach.

4.2 The Interaction Between Risk, Returns and Diversification

One of the key principles of financial markets is that the return on an investment instrument and its underlying risk tend to be closely related. High return prospects are often associated with high risks. Scalable Capital acknowledges this rationale and has constructed its investment universe accordingly, covering asset classes ranging from lower-risk assets such as bonds to higher-risk assets such as emerging market stocks.

However, the interplay between risk and return as such is far more complex than the statement “higher returns require higher risk” might suggest. Looking at longer time periods, we typically observe that assets with a higher average risk generate higher returns. However, if the risk of an asset class deviates from its average or equilibrium risk level, i.e. in the presence – positive or negative – excess risk, returns do not necessarily react proportionally. In fact, the opposite tends to happen: Periods of positive excess risk coincide with poor asset performance and negative excess risk with a strong performance.

In addition, excess risk tends to affect the “synchronicity” with which stock prices move. In phases of positive excess risk the degree of synchronicity typically increases. As a result, common diversification strategies fail in phases of high positive excess risk and can lead to unexpected losses, even for seemingly well-diversified portfolios.

The interaction between risk and return are illustrated in the following sections. For its investment approach, Scalable Capital allows for complex risk-return dynamics observed in reality as well as the fact that the interdependence of assets is too complex in order to be captured by conventional correlation analysis.

4.2.1 Average Risks and Average Returns

Looking at a cross section of asset one finds that assets averaging a higher risk over long periods also tend to yield higher returns in the long run. Figure 10 illustrates the inflation-adjusted average annual returns and risks (measured in terms of volatility) of different asset classes during 1973–2015. All bonds have a volatility below 11% and

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16 Cao and Xu (2010) show that returns of U.S. stocks behave in such a fashion.
4.2 The Interaction Between Risk, Returns and Diversification

Figure 10: Average annual return and volatility levels of different asset classes from 1973–2015.\textsuperscript{17}

inflation-adjusted annual returns below 6%. Stocks, on the other hand, have an average annual return of over 6%, reaching up to 11%, and their corresponding volatilities are over 15% - significantly riskier than the aforementioned bonds.

The indicated regression line in Figure 10 implies that, on average, for each percentage increase in long-term volatility, the average annual return increases by about 0.4%. A look at the development of commodities over the past four decades demonstrates the fact that “high risk equals higher returns” isn’t a universal truth. Even though commodities had a relatively high volatility of close to 20%, their average annual return was only about 3.3%.

4.2.2 Excess Risk and Returns

A longitudinal investigation into how risk fluctuates over the long term for a given asset class paints a different picture of the risk-return relationship. When the risk of, say, equity steeply increases, this usually coincides with below-average or negative market performance. This is especially true when risk levels are already elevated. Figure 11 illustrates the development of the FTSE 100 volatility (measured by the VFTSE, the risk/volatility indicator of the FTSE 100) from January 2006 to March 2016 and highlights the fact that risk can vary drastically. Whilst the volatility had already increased gradually during the subprime crisis, it then rose a further fourfold as a result of the financial crash associated with the fall of Lehman Brothers in mid-September 2008. The national debt crisis of 2011 brought another rise, which increased volatility by a factor of three.

Looking at the FTSE 100 performance in those periods, it is apparent that a severe increase in risk is paralleled by longer term, dramatic market slumps. Namely, roughly 50% and 30% in the course of the banking crisis and the national debt crisis, respectively. The horizontal line in Figure 11 indicates a kind of equilibrium risk level (median) and

\textsuperscript{17}Source: New Frontier Advisors LLC, \url{http://www.fi360.com/fa/help/pdfs/CapitalMarketInputs.pdf}. 
4.2 The Interaction Between Risk, Returns and Diversification

Figure 11: FTSE and FTSE-volatility (as measured by the VFTSE) from 2000 to 2016 (scaled log-scale). Source: Bloomberg

clearly identifies a common theme: When the risk surpasses the median (therefore operating in positive excess risk), we generally observe a plummeting FTSE 100. However, when the excess risk is heavily negative or when the risk level plummets, we can observe that the FTSE 100 tends to increase.

The relationship of the FTSE 100 and the VFTSE, shown in Figure 11 in terms of time series plots, can also be illustrated in terms of a scatterplot as shown in Figure 12. Here too, we can see that increases in volatility are accompanied by losses in the FTSE 100. This phenomenon is independent of the respective level of the FTSE 100 as is indicated by the quasi-parallel stratification of the data.

We can, therefore, infer a systematic relationship between excess risk and share price. Since variations in risks can be measured and modelled, it is possible to keep portfolio risks in check and to help improving overall performance.

The recent Yale-University study of Moreira and Muir (2016) confirms this phenomenon. They compared the performance of U.S. stock portfolios pursuing two different investment strategies. One strategy adopted a typical “Buy & Hold” approach, the other followed a risk-based strategy that decreased exposure during high-volatility periods and increased it during low-risk periods. The authors found that a period of 90 years (1926-2015) the risk-based investment approach delivers better risk-adjusted returns, on average an additional 2% annually, than “Buy & Hold”\textsuperscript{18}. Figure 13 depicts the logarithmic performance of both strategies.

The Yale study further expands the investigation by examining the indexes of 20 OECD countries. There, the authors found that, in 16 out of 20 cases, the risk-based approach achieved a higher Sharpe-Ratio\textsuperscript{19}. Over all 20 countries, the average improve-

\textsuperscript{18} In the Yale study, a difference of 2% p.a. signifies that $1,000 invested in 1926 would be worth about $4,000,000 today, rather than $20,000,000.

\textsuperscript{19} The Sharpe ratio is a common measure to express risk-adjusted performance and is as the average return earned above the risk-free interest rate, divided by a portfolio’s standard deviation. The ratio expresses how effective risk-taking activities were per unit of additional risk assumed.
4.2 The Interaction Between Risk, Returns and Diversification

Figure 12: The interplay of FTSE and VFTSE (log-scale). Source: Bloomberg

The study of Haas and Mittnik (2009) shows that positive excess risks do not only coincide with negative share prices, but are typically also accompanied by correlation surges. The authors investigated the common behaviour of three stock indexes, the S&P 500, FTSE 100 and the DAX, based on so-called “Markov Switching” multi-regime models. Their model assumes that every return observation originates, with a regime-specific probability, from a set of regimes each of which is characterised by a multivariate normal distribution.

Whilst a one-regime model conforms to the common multivariate normal distribution, parameterised by (historical) averages, volatilities and correlations, multi-regime models can approximate non-normalities, such as asymmetry, fat-tails and non-linear dependencies between assets commonly found in financial data, with arbitrary precision by letting the number of regimes grow. Although linear dependencies are postulated within each regime by using correlation as dependence measure, complex dependencies can be captured in terms of local, linear approximations specified by the different regimes.

The mean returns, volatilities and correlations of the indexes for the sample period 1991 - 2001 are shown in Table 4. The data included in Haas and Mittnik (2009) ranges from March 1991 to August 2005 of which the first ten years are used for estimations, while the remaining data is used for validation of the (out-of-sample) predictions. In the following, for the sake of convenience, we only consider the average outcomes for the three indexes (see the last row of Table 4), namely 10.92% for annual returns, 16.03% for annualised volatilities and 0.55 for the correlations. The deviations of the regime-specific estimates in Haas and Mittnik (2009)

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4.2 The Interaction Between Risk, Returns and Diversification

Figure 13: Cumulative performance of portfolios, one following a “Buy & Hold” approach (black), the other a risk-based strategy. Logarithmic scale. Timeframe: 1926-2015. Source: Moreira und Muir (2016, Fig. 3)


<table>
<thead>
<tr>
<th>Index</th>
<th>Annual return</th>
<th>Volat</th>
<th>FTSE</th>
<th>DAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>13.05</td>
<td>14.11</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td>FTSE</td>
<td>7.49</td>
<td>15.29</td>
<td>-</td>
<td>0.60</td>
</tr>
<tr>
<td>DAX</td>
<td>12.22</td>
<td>18.69</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Average</td>
<td>10.92</td>
<td>16.03</td>
<td>0.55</td>
<td></td>
</tr>
</tbody>
</table>

from these averages are presented in Table 5.

In the two-regime case, a high-volatility regime with an increase in volatility of 3.92% leads to a drop in returns of 2.83%. The second regime, a low-risk regime, shows a 3.39% decrease in volatility and is thereby one-third below the average. This reduction in volatility coincides with an annual return increase of 1.87% – compared to the high-risk regime this is an increase in returns of more than half. In periods of high market stress, correlations are 0.10 above and in calmer periods 0.17 below the overall correlation. The discrepancy between correlations in high- and low-risk phases is substantial: 0.65 versus 0.38. With a probability of 60%, the three markets were mainly in calmer waters during the sample period.

As far as the three-regime model is concerned, a similar pattern arises, but with the low-volatility regime of the two component case being split further into moderate and a very low regime, with a volatility drop of -3.01% and -5.96%, respectively. The latter amounts to half of that of the the high-volatility regime. Whilst the returns in both of the higher-volatility components are below the overall average (-1.44% and -1.13%), they experience a substantial surge of 8.75% during low volatility. In addition, the spread of the correlations across the three regimes increases: in high-risk phases it amounts to
4.3 Restrictions on Portfolio Weights

Table 5: Estimation results for multivariate multi-regime models for weekly returns on the S&P 500, FTSE 100 and the DAX. Returns and volatility are annualised. Reported are deviations of the average regime values from the conventionally estimated values given in Table 4. Sample period: March 1991–Feb. 2001. Source: Haas and Mittnik (2009)

<table>
<thead>
<tr>
<th>Number of Regimes</th>
<th>Vola-Regime</th>
<th>Regime-Probability (%)</th>
<th>Vola-Deviation</th>
<th>Return-Deviation</th>
<th>Correlation-Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>High</td>
<td>39.8</td>
<td>3.92</td>
<td>-2.83</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>60.2</td>
<td>-3.39</td>
<td>1.87</td>
<td>-0.17</td>
</tr>
<tr>
<td>3</td>
<td>High</td>
<td>42.5</td>
<td>3.64</td>
<td>-1.44</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>44.1</td>
<td>-3.01</td>
<td>1.13</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>Very Low</td>
<td>13.4</td>
<td>-5.96</td>
<td>8.75</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

0.64, in the lowest-risk regime to only 0.29 - a difference of 0.35.

The results of Haas and Mittnik (2009) not only signify that positive (negative) excess risks tend to coincide with negative (positive) equity returns, they also highlight that, during high-volatility markets, (Markowitz-type) investment approaches systematically overestimate the beneficial efficacy of correlation-based diversification on risk reduction. In other words, when markets turn south, naive correlation-based diversification does not deliver what it promises.

4.3 Restrictions on Portfolio Weights

Apart from downside-risk restrictions in terms of investor-specific VaR-limits, Scalable Capital imposes upper and lower bounds for the weights with which the investment instruments can make up the portfolio. These weight limits vary across risk categories. Such restrictions are common practice in asset management as they avoid excessive concentration and ensure a minimum degree of diversification. In addition to constraining individual instruments, Scalable Capital also imposes group constraints, so that ETFs belonging to similar asset classes have joint upper or lower weight limits. The appropriateness of all limits is continually monitored and may be adjusted as market conditions change.

4.4 The Optimisation Problem

The optimisation problem, which Scalable Capital’s automated investment process solves, reflects the risk-return tradeoff the individual investor faces:

▷ Maximise the reward associated with the expected portfolio return,
▷ penalise for implicit costs arising, for example, from excessive trading and large bid-ask-spreads,
▷ comply with the individual risk tolerance expressed in terms of a 95%-VaR-limit, and
▷ ensure that the weights of the instruments stay within their prescribed bounds.

In mathematical terms, in each period and for each investor $i_k$ in risk category $k$, Scalable Capital solves the following constrained optimisation problem:

$$\max_{w_{i_k}} \mu(w_{i_k}) - p_k(\Delta w_{i_k}), \quad i_k = 1, \ldots, I_k, \quad k = 1, \ldots, K$$
subject to the restrictions
\[ \text{VaR}(w_{ik}) \leq \text{VaR}^*_k, \]  
(4) 
\[ lb_{nk} \leq w_{n_{ik}} \leq ub_{nk}, \quad \text{for all instruments } n = 1, 2, \ldots, N \]  
(5) 
\[ lb_{gk} \leq a'_{gk}w_{g_{ik}} \leq ub_{gk}, \quad \text{for all instrument groups } g = 1, 2, \ldots, G \]  
(6) 
\[ 1'w_{ik} = 1, \]  
(7) 

where:
- \( \mu(\cdot) \): portfolio return after management fee
- \( p_k(\cdot) \): implicit-cost penalty function for risk category \( k \)
- \( w_{ik} \): \( N \)-dimensional vector of weights for portfolio of investor \( i \)
- \( \Delta w_{ik} \): \( N \)-dimensional vector of changes in weights from current weights
- \( \text{VaR}^*_k \): VaR-limit for risk category \( k \)

The attractiveness of MVO-based methods lies in their mathematical simplicity. In fact, mathematical simplicity is the reason for making the restrictive and empirically questionable assumptions that underlie MVO. They do not provide a realistic description of financial markets, they rather give rise to simple solutions that can be found in any standard mathematical toolbox. By not relying on unrealistic assumptions, the solution to Scalable Capital’s optimisation problem (3)–(7) turns out to be considerably less straightforward. Additional computational burden arises from the fact that portfolios are, typically, optimised on a client-specific rather than a risk-class-specific level. As a consequence, the portfolios within a given risk category will be very similar but, in general, not identical. The differences in portfolio weights for two investors in the same risk category can have several causes:

- Different in- and outflow of funds will cause weights to differ.
- If portfolio sizes differ, a lack of fractional investment shares will cause relative weights to differ.
- Different tax situations may result in different allocations.
- If two investors started their investment at different points in time, implicit-cost penalties may produce different weight histories.

It is for these individual differences that Scalable Capital prefers to optimise each client’s portfolio. This implies a substantial computational burden, but is clearly in the best interest of our clients.
5 Portfolio Monitoring and Risk-based Rebalancing

As financial markets evolve, a portfolio’s allocation can become suboptimal and, thus, ought to be rebalanced. Common portfolio rebalancing strategies are calendar-driven. They simply re-adjust the portfolio after a fixed time interval and re-set all weights to preassigned, fixed values. Scalable Capital takes a more intelligent, event-driven approach. We continually monitor each investor’s portfolio, to make sure that it is in line with the designated risk level and that all weight limits are satisfied. If a risk projection signals a violation of the VaR-target, the portfolio will be re-optimised to ensure that it stays within the assigned risk corridor – regardless of when the last rebalancing took place. This allows Scalable Capital to immediately respond to changes in market conditions without being subjected to calendrical rituals.

![Projected Risk After Rebalancing](image)

Figure 14: A projected risk violation (left) triggers a rebalancing of the portfolio. Risk projection after the rebalancing step (right).

5.1 Portfolio Monitoring

If market constellations change, a portfolio may not perform as expected. Scalable Capital’s major concern is that the projected portfolio risk may exceed the investor’s risk limit. In such a circumstance, the portfolio will be adjusted to lower the risk exposure. Similarly, if overall market risk subsides, a portfolio can become overly conservative and, as a consequence, is likely to underperform. In this case, portfolio weights are shifted towards assets with higher risk and return potential. Scalable Capital regularly monitors all client portfolios and examines whether risk projections and risk targets are in line or diverge.

5.2 Risk-based Rebalancing

If the risk projection is outside the designated risk corridor, the portfolio will be re-optimised so that it is in line with the investor’s risk profile. The need for rebalancing may also arise when upper or lower portfolio weight limits are violated because asset prices have moved over time.

The risk-based rebalancing principle is demonstrated in Figure 14. The left graph illustrates a portfolio projection generated during a monitoring step which signals that the risk projection (red line) violates the designated one-year risk target (green area).
5.2 Risk-based Rebalancing

This automatically triggers the rebalancing routine, which – in the case of risk overshooting – re-adjusts the portfolio weights in an optimal fashion to a more conservative composition. The risk projection for the rebalanced portfolio, shown in the right graph, is now in line with the investor’s risk profile. Typically, this goes together with lowering the portfolio’s upside potential, as is reflected in Figure 14 by a lower path for the expected performance (dashed line).

Apart from risk-contingent re-allocation, portfolio adjustments can also be made in response to a violation of predefined risk weights for individual or groups of asset classes, or when deposits, withdrawals or changes are made to the client’s account.
6 Regular Reviews and Updating Client Profiles

Investors’ financial statuses and personal situations are likely to change over time. Such changes, and the fact that investment goals or risk attitudes are not carved in stone, may require an update of the investor profile and, possibly, a realignment of the investment strategy. Regular reviews of client profiles are therefore part of the Scalable Capital investment process. Once an investor reports changes to his or her profile, Scalable Capital will examine to what extent investor-specific parameters need to be adapted. Scalable Capital will regularly inform their investors about their profile currently on record and ask them to verify and, if needed, to update their profile. Such updates will automatically trigger a re-evaluation of the appropriateness of the investor’s risk classification and may – after consultation with the client – lead to an adjustment of the investment strategy.

Regulation and investment practices are perpetually changing. Scalable Capital systematically monitors such changes and may respond by adapting its processes to be inline with new regulation or investment practices.
References


